SOLVED EXAMPLES

[FIRST SOLVE THE QUESTIONS YOURSELF, THEN GO THROUGH THE SOLUTIONS]

Ex. 1 The capacitor of capacitance $4\mu F$ and $6\mu F$ are connected in series. A potential difference of 500 volts is applied to the outer plates of the two capacitor system. Then the charge on each capacitor is numerically-

(A) 6000 C (B) 1200C

(C) 1200 μC (D) $6000\mu C$

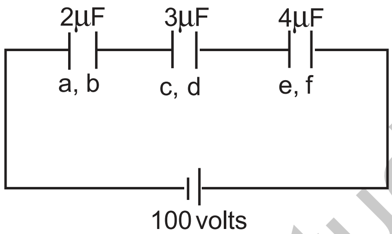
Sol. $C_R = \dfrac{C_1C_2}{C_1+C_2} = 2.4\ \mu F$

Charge flown through the circuit

$= 2.4 \times 500 \times 10^{-6} C = 1200\ \mu C$

Ans.[C]

Ex. 2 Three capacitors are connected to D.C. source of 100 volts as shown in the adjoining figure. If the charge accumulated on plates of C_1, C_2 and C_3 are q_a, q_b, q_c, q_d, q_e and q_f respectively, then



(A) $q_b + q_d + q_f = \dfrac{100}{2}$ C

(B) $q_b + q_d + q_f = 0$

(C) $q_a + q_c + q_e = 50$ C

(D) $q_b = q_d = q_f$

Sol. In series combination, charge is same on capacitor. **Ans.[D]**

Ex.3 Three capacitors each of capacitance $1\mu F$ are connected in parallel. To this combination, a fourth capacitor of capacitance $1\mu F$ is connected in series. The resultant capacitance of the system is-

(A) $4\mu F$ (B) $2\mu F$

(C) 4/3 μF (D) 3/4 μF

Sol. Resultant of parallel combination

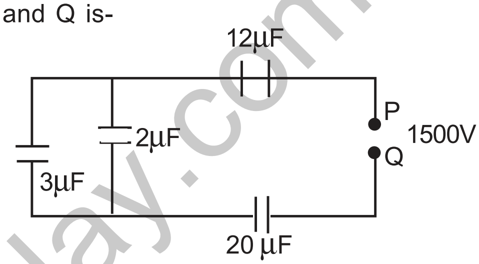
$= 3 \times 10^{-6}\ \mu F$

Total capacitance of combination is

$$\frac{1}{C_r} = \frac{1}{3\times10^{-6}} = \frac{4}{3} \times 10^6$$

$\Rightarrow C_r = \dfrac{3}{4}\ \mu F$ **Ans.[D]**

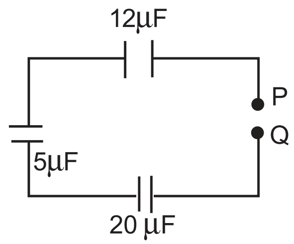
Ex. 4 In the circuit diagram shown in the adjoing figure, the resultant capacitance between P and Q is-



(A) 47 μF (B) $3\mu F$

(C) 60 μF (D) 10 μF

Sol. The given circuit can be drawn as



Where $C = (3 + 2)\ \mu F = 5\mu F$

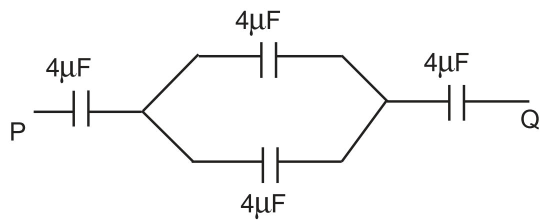
$$\frac{1}{C_{PQ}} = \frac{1}{5}+\frac{1}{20}+\frac{1}{12} = \frac{20}{60} = \frac{1}{3}$$

$\Rightarrow C_{PQ} = 3\mu F$ **Ans.[B]**

Ex.5 Four condensers each capacity $4\mu F$ are connected as shown in figure $V_P - V_Q = 15$ volts. The energy stored in the system is-

(A) 2400 ergs (B) 1800ergs

(C) 3600 ergs (D) 5400 ergs



Sol. Total capacitance of given system]

$$\frac{1}{C_R} = \frac{5}{8} \Rightarrow C_R = \frac{8}{5}\mu F$$

$$\text{Energy stored} = \frac{1}{2}C_R V^2$$

$$= \frac{1}{2} \times \frac{8}{5} \times 10^{-6} \times 225$$

$$= 180 \times 10^{-6} \text{ joule}$$

$$= 180 \times 10^{-6} \times 10^{7} \text{ ergs}$$

$= 1800$ ergs **Ans.[B]**

Ex.6 Two capacitors each of 0.5 µF capacitance are connected in parallel and are then charged by 200 volts. D.C. supply. The total energy of their charges (in joules) is-

(A) 0.01 (B) 0.02

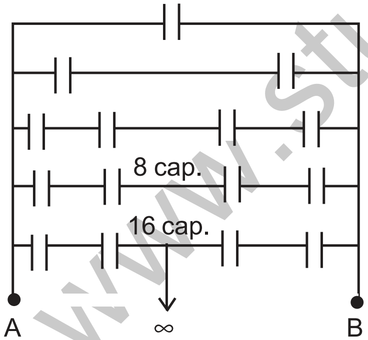
(C) 0.04 (D) 0.06

Sol. $C_R = 1\mu F$, $E_R = \frac{1}{2}C_R V^2$

$$\Rightarrow E_R = \frac{1}{2} \times 1 \times 10^{-6} \times 200 \times 200$$

$= 2 \times 10^{-2} = 0.02J$ **Ans.[B]**

Ex.7 An infinite number of identical capacitors each of capacitance 1µF are connected as in adjoining figure. Then the equivalent capacitance between A and B is-



(A) 1µF (B) 2µF

(C) 1/2 µF (D) ∞

Sol. This combination forms a G.P.,

$$\therefore \text{ Sum} = S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\ldots.$$

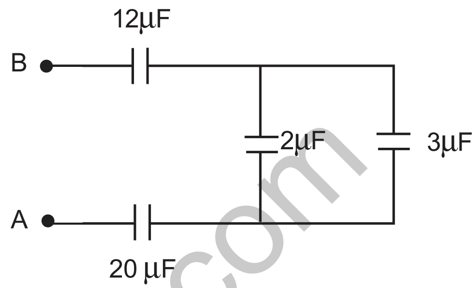
Sum of infinite G.P. is $S = \frac{a}{1-r}$

Here a = first term = 1 and r = common ratio $= \frac{1}{2}$

$$\Rightarrow S = \frac{1}{1-1/2} = 2 \quad \therefore C_R = 2\mu F$$

Ans.[B]

Ex.8 Four capacitors are connected as shown in the adjoining figure. The potential difference between A and B is 1500 volts. The energy stored in 2µF capacitance will be-



(A) 5.06J (B) 0.506J

(C) 50.6J (D)none of the above.

Sol. $C = 2 + 3 = 5\mu F$

$$V_{AB} = \frac{q}{12} + \frac{q}{20} + \frac{q}{5} = \frac{q}{3}$$

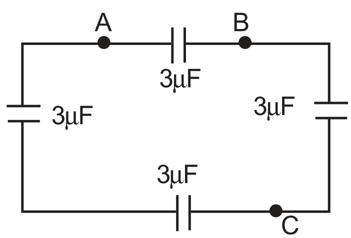
$$\Rightarrow 1500 = \frac{q}{3} \Rightarrow q = 4500 \ \mu C$$

Hence potential difference across 2µF capacitor $= \frac{4500}{5} = 900$ V

$$\text{Energy stored} = \frac{1}{2} \times 2 \times 10^{-6} (900)^2 = 0.81 \text{ J}$$

Ans.[D]

Ex.9 Four capacitors of each capacity 3µF are connected as shown in the adjoining figure. The ratio of equivalent capacitance between A and B and between A and C will be-



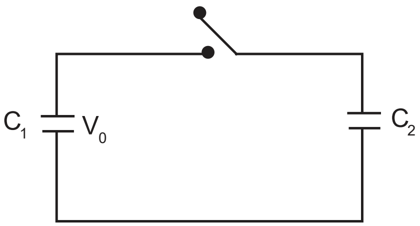
(A) 4 : 3 (B) 3 : 4

(C) 2 : 3 (D) 3 : 2

Sol. $C_{AB} = 3 + \frac{3}{3} = 4$, $C_{AC} = \frac{3}{2} + \frac{3}{2} = 3$

$\therefore C_{AB} : C_{AC} = 4 : 3$ **Ans.[A]**

Ex.10 A capacitor of capacity C_1 is charged to the potential of V_0. On disconnecting with the battery, it is connected with a capacitor of capacity C_2 as shown in the adjoining figure. The ratio of energies before after the connection of switch S will be-



(A) $(C_1 + C_2)/C_1$ (B) $C_1 / (C_1 + C_2)$
(C) C_1C_2 (D) C_1/C_2

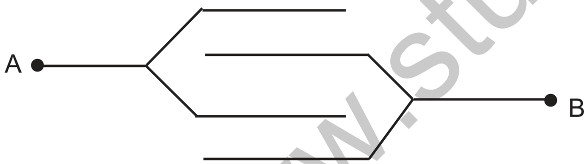
Sol. Energy $= \dfrac{q^2}{2C}$

$$\Rightarrow \frac{E_{before}}{E_{after}} = \frac{q^2/2C_1}{q^2/[2(C_1+C_2)]}$$

($\because$ q remains same)

$$\Rightarrow \frac{E_B}{E_A} = \frac{C_1+C_2}{C_1}$$ **Ans.[A]**

Ex.11 Four plates of the same area of cross-section are joined as shown in the figure. The distance between each plate is d. The equivalent capacity across AB will be-

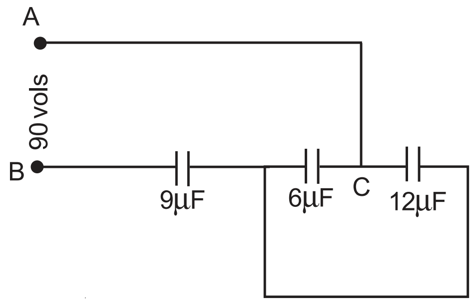


(A) $\dfrac{2\varepsilon_0 A}{d}$ (B) $\dfrac{3\varepsilon_0 A}{d}$
(C) $\dfrac{3\varepsilon_0 A}{2d}$ (D) $\dfrac{\varepsilon_0 A}{d}$

Sol. The arrangement shown in the figure is equivalent to three capacitors in parallel hence resultant capacitance = $\dfrac{3\varepsilon_0 A}{d}$ **Ans.[B]**

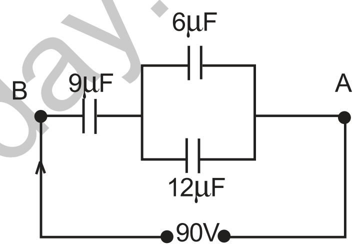
Ex.12 The capacity of the capacitors are shown in the adjoining fig. The equivalent capacitance between the points A and B and the charge on the 6μF capacitor will be-

(A) 27μF, 540μC (B) 15μF, 270μC
(C) 6μF, 180 μC (D) 15μF, 90μC



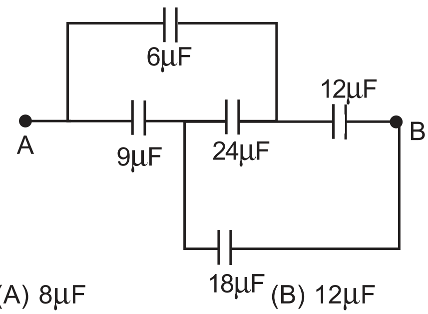
Sol. $C_{eq} = \dfrac{9\times18}{9+18} = 6\mu F$

$V_{AC} \cdot C_{AC} = C_{CB} \cdot V_{CB}$
and $V_{AC} + V_{CB} = 90$
$\therefore V_{AC} = 30V$,
hence $q_6 = 30 \times 6 = 180\mu C$



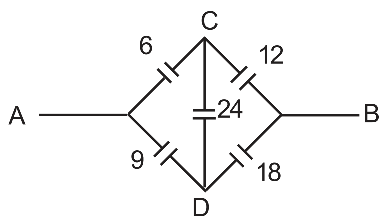
Ans.[C]

Ex.13 In the connection shown in the adjoining figure. the equivalent capacity between A and B will be-



(A) 8μF (B) 12μF
(C) 20μF (D) 2 μF

Sol. Given circuit can be drawn as



Here C , D are equipotential points

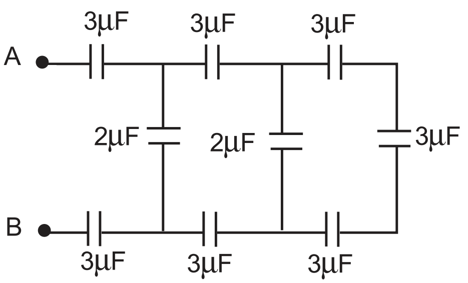
$\therefore$ 24 μF capacitor holds no charge as

$$\Delta V = 0$$

$$\therefore C_{eq} = \frac{12\times6}{12+6} + \frac{9\times18}{9+18} = 10\mu F$$

Ans.[D]

Ex.14 The resultant capacitance between A and B the following figure is equal to-



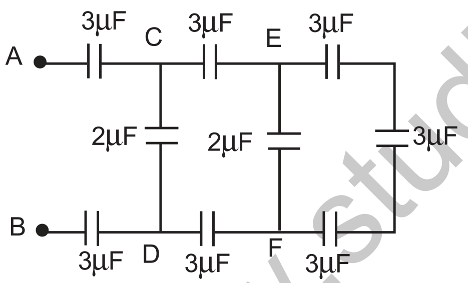
(A) 1μF (B) 3μF

(C) 2μF (D) 1.5 μF

Sol. Total series capacitance across EF = 3μF

Capacitance across EF = 2 + 1 = 3μF

Thic capacitance is in series with 3μF capacitances at CD.



Hence total series capacitance

$$\frac{1}{C'} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1\ ;$$

$\therefore$ C' = 1μF

This capacitance is in parallel with capacitance of 2μ across CD.

Hence $C_{T\ (AB)} = 2 + 1 = 3\mu F$

This capacitance is in series with 3μF capacitances across AB. Hence total capacitance across AB.

$$\frac{1}{C''} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1\ ;$$

$\therefore$ C'' = 1μF

Ans.[A]

Ex.15 Two condensers of capacities 2C and C are joined in parallel and charged upto potential V. The battery is removed and the condenser of capacity C is filled completely with a medium of dielectric constant K. The p.d. across the capacitor will now be-

(A) $\frac{3V}{K+2}$ (B) $\frac{3V}{K}$

(C) $\frac{V}{K+2}$ (D) $\frac{V}{K}$

Sol. $q_1 = 2CV$, $q_2 = CV$

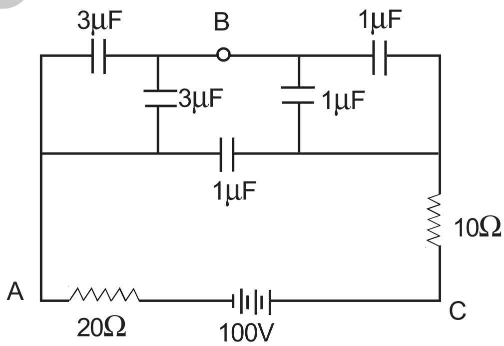
Now condenser of capacity C is filled with dielectric K, therefore $C_2 = KC$

As charge is conserved,

$\therefore q_1 + q_2 = (C_2 + 2C)\ V$

$$\Rightarrow V' = \frac{3CV}{(K+2)C} = \frac{3V}{K+2}$$ **Ans.[A]**

Ex.16 In the figure below, what is the potential difference between the point A and B and between B and C respectively in steady state-



(A) V_{AB} = 100volts , V_{BC} = 100 volts

(B) V_{AB} = 75 volts , V_{BC} = 25volts

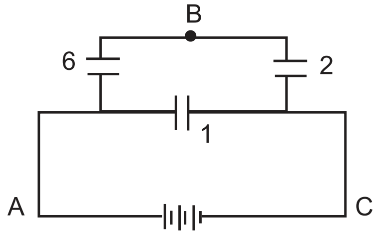
(C) V_{AB} = 25volts , V_{BC} = 75volts

(D) V_{AB} = 50Volts , V_{BC} = 50volts

Sol. $$C_{equivalent} = \frac{(3+3)\times(1+1)}{(3+3)+(1+1)} + 1$$

$$= \frac{(6\times2)}{6+2} + 1 = \frac{12}{8} + 1 = \frac{5}{2}\mu F$$

$$\therefore Q = C \times V = \frac{5}{2} \times 100 = 250\mu C$$



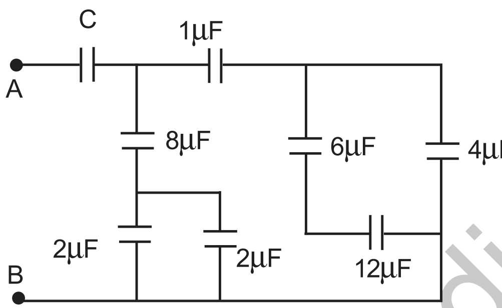
Charge in 6μF branch = VC

$= \left(\frac{6\times2}{6+2}\right) \times 100 = \frac{12\times100}{8} = 150\ \mu C$

$\therefore V_{AB} = \frac{150}{6} = 25V$

and $V_{BC} = 100 - V_{AB} = 75V$ **Ans.[C]**

Ex.17 In the following circuit, the resultant capacitance between A and B is 1μF. Then value of C is &



(A) $\frac{32}{11}$ μF (B) $\frac{11}{32}$ μF

(C) $\frac{23}{32}$ μF (D) $\frac{32}{23}$ μF

Sol. 12μF and 6μF are in series and again are in parallel with 4μF.

Therefore resultant of these three will be

$= \frac{12\times6}{12+6} + 4 = 8\mu F,$

This equivalent system is series with 1μF , its equivalent capacitance

$= \frac{8\times1}{8+1} = \frac{8}{9}\ \mu F$ .....(1)

Equivalent of 8μF, 2μF and

$2\mu F = \frac{4\times8}{4+8} = \frac{32}{12} = \frac{8}{3}\mu F$ .....(2)

(1) and (2) are in parallel and are in series with C.

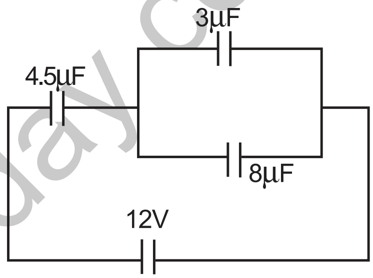
$\therefore \frac{8}{9}+\frac{8}{3} = \frac{32}{9}$ and $C_{eq} = 1 = \frac{\frac{32}{9}\times C}{\frac{32}{9}+C}$

$\Rightarrow \frac{32}{9} + C = \frac{32}{9} C$

$\Rightarrow \frac{32}{9} C - C = \frac{32}{9}$

$\Rightarrow C = \frac{32}{9} \times \frac{9}{23} = \frac{32}{23}\mu F$ **Ans.[D]**

Ex.18 In the circuit shown in the figure, the potential difference across the 4.5μF capacitor is-



(A) $\frac{8}{3}$ volts (B) 4 volts

(C) 6 volts (D) 8 volts

Sol. Total capacity of the combination

$\frac{1}{C} = \frac{10}{45} + \frac{1}{9} = \frac{10+5}{45} = \frac{1}{3}$

$\Rightarrow C = 3\mu F$

The charge through the circuit

$= 3 \times 12 = 36\mu C$

$\therefore$ Potential difference across 4.5μF

capacitor $= \frac{q}{C} = \frac{36}{4.5} = 8$ volts **Ans.[D]**

Ex.19 A capacitor 4 μF charged to 50V is connected to another capacitor of 2μF charged to 100V with plates of like charges connected together. The total energy before and after connection in multiples of (10^{-2} J) is-

(A) 1.5 and 1.33 (B) 1.33 and 1.5
(C) 3.0 and 2.67 (D) 2.67 and 3.0

Sol. The total energy before connection

$= \frac{1}{2} \times 4 \times 10^{-6} \times (50)^2 + \frac{1}{2} \times 2 \times 10^{-6} \times (100)^2$

$= 1.5 \times 10^{-2}$ J

When connected in parallel

$= 4 \times 50 + 2 \times 100 = 6 \times V \Rightarrow V = \frac{200}{3}$

Total energy after connection

$= \frac{1}{2} \times 400 \times 10^{-6} \times \frac{200}{3} = 1.33 \times 10^{-2}$ J

Ans.[A]

Ex.20 Two capacitors of 3pF and 6pF are connected in series and a potential difference of 5000V is applied across the combination. They are then disconnected and reconnected in parallel. The potential between the plates is-

(A) 2250V (B) 1111V
(C) 2.25 x10^6V (D) 1.1 x 10^6 V

Sol. $\frac{1}{C} = \frac{1}{3} + \frac{1}{6} \Rightarrow C = 2$pF

Total charge $= 2 \times 10^{-12} \times 5000 = 10^{-8}$ C

The new potential when the capacitors are connected in parallel is

$$V = \frac{10^{-8}}{(3+6)\times10^{-12}} = 1111\text{V}$$

Ans.[B]

Ex.21 Three condensers are connected in series across a 75 volt supply. The voltages across them are 20, 25 and 30 volts respectively and the charge on each is 3 x 10^{-3} C. Find the capacity of each condenser and also the combination.-

Sol. Here Q = 3 × 10^{-3} C

V = 70V , V_1 = 20 V , V_2 = 25V , V_3 = 30V

Let C_1 , C_2 and C_3 be the capacities of the condensers respectively and c be the capacity of the combination.

since $C = \frac{Q}{V}$

$\therefore C_1 = \frac{3\times10^{-3}}{20} = 1.5 \times 10^{-4}$ F

$= 15 \times 10^{-5}$ F

$C_2 = \frac{3\times10^{-3}}{25} = 1.2 \times 10^{-4}$ F

$= 12 \times 10^{-5}$ F

$C_3 = \frac{3\times10^{-3}}{30} = 1.0 \times 10^{-4}$ F $= 10 \times 10^{-5}$ F

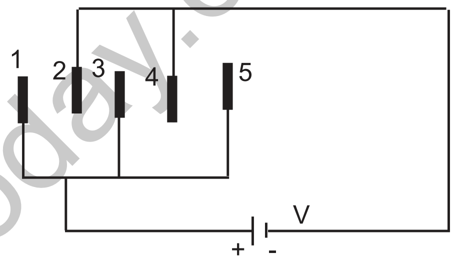
Total capacity of the combination

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \left[\frac{10^5}{15} + \frac{10^5}{12} + \frac{10^5}{10}\right] \text{ or } \frac{1}{C} = \frac{45}{180} \times 10^5$$

or C = 40μF or C = 40 × 10^{-6} F

Ex.22 Five identical plates each of area A are joined a shown in the figure. The distance between the plates is d. The plates are connected to a p.d. of V volts. The charge on plates 1 and 4 will be-



(A) $\frac{\varepsilon_0 AV}{d}, \frac{2\varepsilon_0 AV}{d}$ (B) $\frac{-\varepsilon_0 AV}{d}, \frac{2\varepsilon_0 AV}{d}$

(3) $\frac{\varepsilon_0 AV}{d}, \frac{-2\varepsilon_0 AV}{d}$ (D) $\frac{-\varepsilon_0 AV}{d}, \frac{-2\varepsilon_0 AV}{d}$

Sol. The capacity C_{12} of the parallel plate capacitor formed by 1 and 2 is given by

$$C_{12} = \frac{\varepsilon_0 A}{d}$$

Charge on plate 1, $q_1 = C_{12}V = \frac{\varepsilon_0 AV}{d}$

Since the plate 1 is connected to +ve terminal the source of emf, therefore charge q_1 is +ve. The capacity C_{23} of the parallel plate capacitor formed by plates 2 & 3 is given by $C_{23} = \frac{\varepsilon_0 A}{d}$

Similarly, the capacity C_{34} of the parallel plate capacitor formed by plates 3 and 4 is given by $C_{34} = \frac{\varepsilon_0 A}{d}$

It may be noted here that the plates 1 & 3 constitute no capacitor because these plates are connected together.

Since capacitors of capacities C_{23} & C_{34} are connected in parallel, therefore the net capacitance between the plates 1 and 4 is given by $C_{14} = \dfrac{2\varepsilon_0 A}{d}$

Charge on plate 4, $q_1 = \dfrac{2\varepsilon_0 A}{d} V$

The charge on plate 4 is negative because the plate is connected to the negative terminal of the source of emf. **Ans.[C]**

Ex. 23 The capacities of two conductors are C_1 and C_2 and their respective potentials are V_1 and V_2. If they are connected by a thin wire, then the loss of energy will be given by-

(A) $\dfrac{C_1C_2(V_1+V_2)}{2(C_1+C_2)}$ (B) $\dfrac{C_1C_2(V_1-V_2)}{2(C_1+C_2)}$

(C) $\dfrac{C_1C_2(V_1-V_2)^2}{2(C_1+C_2)}$ (D) $\dfrac{(C_1+C_2)(V_1-V_2)}{C_1C_2}$

Sol. (i) Looking directly at dimensions (C) is correct.

(ii) As total charge conserves, therefore charge redistributes such that both the capacitors are finally the same potential

$\therefore (C_1 + C_2)V = C_1V_1 + C_2V_2$ = Total charge

$$\Rightarrow \quad V = \frac{C_1V_1+C_2V_2}{C_1+C_2}$$

$$\Rightarrow \Delta E = \frac{1}{2}(C_1 + C_2)\left[\frac{C_1V_1+C_2V_2}{C_1+C_2}\right]^2 - \frac{1}{2}C_1V_1^2 - \frac{1}{2}C_2V_2^2$$

$$\Rightarrow \Delta E = \frac{1}{2}\left[\frac{C_1^2V_1^2+C_2^2V_2^2+2C_1C_2V_1V_2}{(C_1+c_2)} - \frac{C_1^2V_2^2-C_1C_2V_1^2-C_1C_2V_2^2}{(C_1+C_2)}\right]$$

$$= \frac{-C_1C_2(V_1-V_2)^2}{2(C_1+C_2)}$$

$\therefore$ The final energy is lesser than the initial energy stored in the capacitors.

Ans.[C]

Ex. 24 A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved further apart by means of insulating handles, then-

(A) The change on the capacitor increases

(B) The voltage across the plates decreases

(C) The capacitance increases

(D) The electrostatic energy stored in he capacitor increases

Sol. When the battery is disconnected, the charge will remains same in any case.

Capacitance of a parallel plate capacitor is given by $C = \dfrac{\varepsilon_0 A}{d}$

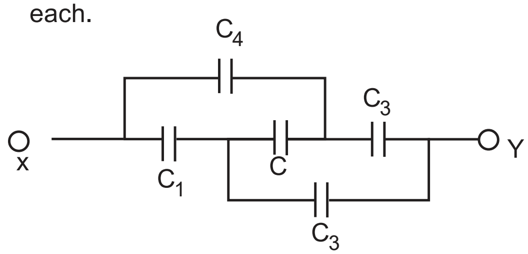
When d is increased, capacitance will decreases and because the charge remains the same. So hence the electrostatic energy stored in the capacitor will increases.

Change in stored energy $= \dfrac{1}{2}Q\,(V_2 - V_1)$

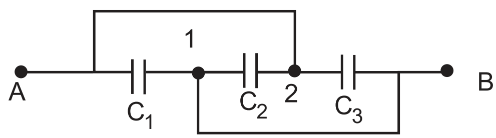
$= +$ ve ($\because$ We know that $V_2 > V_1$)

Ans.[D]

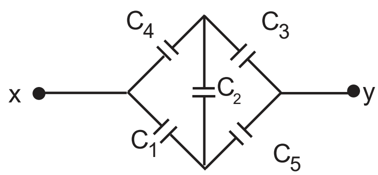
Ex.25 **(a)** Find the effective capacitance between points X and Y in the given fig. Assuming that $C_2 = 10\mu F$ and other capacitors are $4\mu F$ each.



(b) Find the capacitance of a system of identical capacitors between points A and B shown in fig



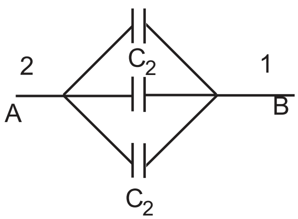
Sol. **(a)** The circuit is redrawn. As the two arms are balanced, no current flows through



C_2, C_3 and C_4 are in series, hence their equivalent capacitance = 2μF

Similarly the equivalent capacitance of C_1 and C_5 = 2μF. Corresponding to points X and Y there two are in a parallel combination. Hence the effective capacitance between X and Y is 2 + 2 = 4μF

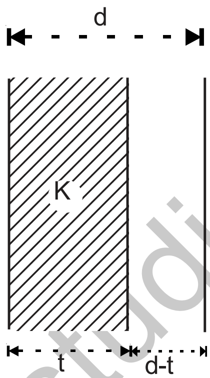
(b) The arrangement of capaction shown in fig is equivalent to the arrangement show in fig.



The arrangement is connected in paralle. Hence equivalent capacitance C is given by

$$C = C_1 + C_2 + C_3$$

Ex.26 A parallel plate condenser consists of two metal plates of area A and separation d. A slab of thickness t and dielectric constant K is inserted between the plates with its faces parallel to the plates and having the same surface are as that of the plates. Find the capacitance of the system. if K = 2, for what value of t/d will the capacitance of the system be 3/2 times that of the air condenser alone ? Calculate the energy in the two cases and account for the energy change.-



Sol. We know that the capacitance of a capacitor with a dielectric of

dielectric constant K is given by $\frac{\varepsilon_0 KA}{d}$

Hence the capacitance C_1 of the capacitor with dielectric constant K is given by

$C_1 = \frac{\varepsilon_0 KA}{t}$ where t is its thickness

The capacitance C_2 of remaining capacitor is

$$C_2 = \frac{\varepsilon_0 A}{(d-t)}$$

Effective capacitance C of C_1 and C_2 is given by $C = \frac{C_1 C_2}{(C_1 + C_2)} = \frac{\varepsilon_0^2 A^2 K / t(d-t)}{\frac{\varepsilon_0 KA}{t} + \frac{\varepsilon_0 A}{(d-t)}}$

On simplification, we have] $C = \frac{\varepsilon_0 A}{d-(t/2)}$

The capacitance C_a of air condenser with thickness d is $C_a = \frac{\varepsilon_0 A}{d}$

Now $C = \frac{3}{2} C_a$ or $\frac{\varepsilon_0 A}{d-(t/2)} = \frac{3}{2}\frac{\varepsilon_0 A}{d}$

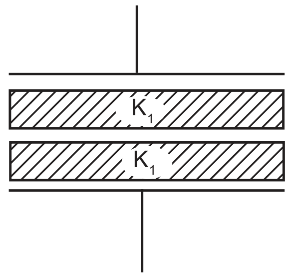
Solving we have $\frac{t}{d} = \frac{2}{3}$

If q = charge on the condenser (remains unchanged), then initial energy E_i in the air condenser = $(q^2/2C_a)$ and final energy E_f after introducing dielectric = $(q^2/2C)$

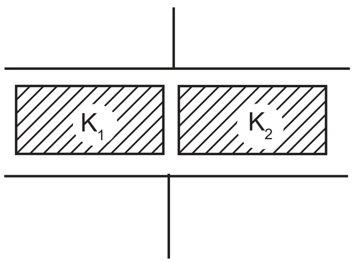
$$\therefore \frac{E_i}{E_f} = \frac{3}{2}$$

When a dielectric is intorduced, it decreases the potential energy of the condenser. The loss is used to polarise the dielectric.

Ex.27 **(a)** Two dielectric slabs of dielectric constant K_1 and K_2 have been filled in between the plates of a capacitor as shown in fig. What will be the capacitance in each case.



(b) A capacitor if filled with two dielectric of same dimensions but of dielectric constant 2 and 3 respectively. Find the ratio of the capacitances in the two possible arrangements



Sol. **(a)** Let A be the area of each plate of the capacitor and d be the distance between the two plates. If the capacitance be C_1 and C_2 respectively, then $C_1 = \varepsilon_0 \frac{K_1 A/2}{d}$ and

$$C_2 = \varepsilon_0 \frac{K_2 A/2}{d}$$

Let C be the equivalent capacitance, then C

$$= C_1 + C_2 = \varepsilon_0 \frac{K_1 A}{2d} + \varepsilon_0 \frac{K_2 A}{2d}$$

$$= \frac{\varepsilon_0 A}{2d} (K_1 + K_2)$$

{∴ Two condensers are in parallel}

The arrangement shown in fig is equivalent to two capacitors joined in series. Let their capacitance be C_1 and C_2 respectively. The

$$C_1 = \varepsilon_0 \frac{K_1 A}{d/2} \text{ and } C_2 = \varepsilon_0 \frac{K_2 A}{d/2}$$

$$\text{Now } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2\varepsilon_0 K_1 A} + \frac{d}{2\varepsilon_0 K_2 A}$$

$$= \frac{d}{2\varepsilon_0 A}\left[\frac{K_2 + K_1}{K_1 K_2}\right]$$

$$C = \frac{2\varepsilon_0 A}{d}\left[\frac{K_1 K_2}{K_2 + K_1}\right] \quad .....(2)$$

(b) The ratio of capacitances in the two possible arrangement is given by [from eqs. (1) and (2)]

$$= \frac{\frac{2\varepsilon_0 A}{2d}(K_1 + K_2)}{\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)} = \frac{(K_1 + K_2)^2}{4K_1 K_2} = \frac{(2+3)^2}{4 \times 2 \times 3}$$

= 25 : 24 **Ans.**

Ex.28 The capacitance of a parallel plate capacitor is 400 pico farad and its plates are separated by 2mm in air (i) What will be the energy when it is charged to 1500 volts. (ii) What will be the potential difference with the same charge if plate separation is doubled ? (iii) How much energy is needed to doubled the distance between its plates?

Sol. Here C = 400 pico-farad = 400 x 10^{-12} farad and d = 2mm = 0.002 metere

(i) The capacitor is charged to a potential V = 1500 volts. The energy W of the capacitor is given by

$$W = \frac{1}{2} CV^2 = \frac{1}{2} \times (400 \times 10^{-12})(1500)^2$$

$$= 4.5 \times 10^{-4} \text{ Joules}$$

(ii) We know that the capacity of parallel plate condenser C = ε_0A/d. When d is doubled, the new capacity. C' becomes halved

$$\text{i.e. } C' = \frac{1}{2} C = \frac{1}{2} \times 400 \times 10^{-12}$$

$$= 200 \times 10^{-12} \text{ farad}$$

Charge on the capacitor

$q = CV = 400 \times 10^{-12} \times 1500$

$= 6 \times 10^{-7}$ coul.

Let the new potential difference be V' then for the same charge q, we have

$q = C'V' = 6 \times 10^{-7}$ or $200 \times 10^{-12} \times V'$

$= 6 \times 10^{-7} \Rightarrow V' = 3000$ volts

(iii) The energy required to double the distance between the plate = Final energy - initial energy = $\frac{1}{2}C'V'^2 - \frac{1}{2}CV^2$

$$= \frac{1}{2}(200 \times 10^{-12})(3000)^2 - (4.5 \times 10^{-4})$$

$= 9 \times 10^{-4} - 4.5 \times 10^{-4}$

$= 4.5 \times 10^{-4}$ joules **Ans.**

Ex.29 A condenser has capacitance 10 micro - farad and is charged to a potential 150 V. Calculate the charge. A second condenser has a capacitance of 20 micro farad and is charged t a potential of 300 volts. If after charging, the two condensers are connected in parallel by wires of negligible capacitance, how much energy is dissipated ?

Sol. Charge on 10μF condenser

$= 10 \times 10^{-6} \times 150 = 1.5 \times 10^{-3}$ C

Charge on 20μF condenser

$= 20 \times 10^{-6} \times 300C = 6 \times 10^{-3}$ C

When the two condensers are connected in parallel, the total charge

$= (1.5 \times 10^{-3} + 6 \times 10^{-3}) = 7.5 \times 10^{-3}$ C

If V be the potential, then 30×10^{-6} V

$= 7.5 \times 10^{-3}$

$$V = \frac{7.5 \times 10^{-3}}{30 \times 10^{-6}} = 250 \text{ volts}$$

Energy of first condenser before connection

$= 1/2 \times (1.5 \times 10^{-3})\ 150$

Energy of second condenser before connection $= 1/2 \times (6 \times 10^{-3})\ 300$

Totally energy of condensers before connection

$$= \frac{1}{2}(1.5 \times 10^{-3})\ 150 + \frac{1}{2}(6 \times 10^{-3})\ 300$$

= 1.0125J

Energy after connection

$$= \frac{1}{2} \times (7.5 \times 10^{-3})\ 250 = 0.9375\ J$$

Energy dissipated = 1.0125 - 0.9325 = 0.075 J

Ans.

Ex.30 A parallel plate capacitor of plate area $A = 10^{-2}$ metere^2 metre is charged to $V_0 = 100$ volts. Then after removing the charging battery, a slab of insulating material of thickness $b = 0.5 \times 10^{-2}$ metre and dielectric constant K = 7 is inserted between the plates. Calculate the free charge on the plates of the capacitor, electric field intensity in air, electric field intensity in the dielectric potential difference between the plates and capacitance (with dielectric present, if the distance between plates is 1 cm.

Sol. The capacitance C_0 before the slab is introduced

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.9\times10^{-12})(10^{-2})}{10^{-2}}$$

$= 8.9 \times 10^{-12}$ farad

$\therefore$ Free charge] $q = C_0V_0$

$= (8.9 \times 10^{-12}) \times 100 = 8.9 \times 10^{-10}$ C

Now, electric field intensity

$$E_0 = \frac{100}{10^{-2}} = 1.0 \times 10^4 \text{ volt/metre.}$$

Electric field intensity in dielectric

$$E = \frac{E_0}{K} = \frac{1.0\times10^4}{7} = 1.43 \times 10^3 \text{ volt/metre}$$

Potential difference between the plates with dielectric present is given by

$V = E_0\ (d - b) + Eb$

$= (1.0 \times 10^4)\ (10^{-2} - 0.5 \times 10^{-2})$

$+ (1.43 \times 10^3)\ (0.5 \times 10^{-2}) = 57$ volts

The free charge on the plate is the same as before. The capacitance with dielectric present is

$$C = \frac{q}{V} = \frac{8.9\times10^{-10}\,C}{57\,V} = 16 \times 10^{-12} \text{ farad}$$

$= 16\mu\mu F$ **Ans.**

Ex.31 The plates of a parallel plate capacitor are 1 cm apart and 2m^2 in area. A potential difference of 6000 volts is applied across the capacitor. When a sheet of dielectric is inserted between the plates ; the potential difference decreases to 2000 volts. Calculate (i) the original capacitance C_0, (ii) the charge Q on each plate, (iii) the capacitance C after insertion of the dielectric (iv) the dielectric coefficient K of the dielectric, (v) the permitivity ε of the dielectric, (vi) the induced charge Q_1 on each face of the dielectric, (vii) the original electric intensity E_0 between the plates, (viii) the electric intensity after insertion of the dielelctric-

Sol. (i) The original capacitance

$C_0 = \varepsilon_0 A/d$

$$\therefore\ C_0 = \frac{(8.85\times10^{-12})}{1\times10^{-2}} \times 2$$

$= 1.77 \times 10^{-9}$ farad

(ii) The charge Q on each plate = CV

$\therefore\ Q = 1.77 \times 10^{-9} \times 6000$

$= 1.062 \times 10^{-5}$ coulomb

(iii) When the dielectric is inserted, the new capacitance becomes

$$C_1 = \frac{Q}{V} = \frac{1.062\times10^{-5}}{2000} = 5.31 \times 10^{-9} \text{ farad}$$

(iv) We know that dielectric constant

$$K = \frac{C_1}{C_0} = \frac{5.31\times10^{-9}}{1.77\times10^{-9}} = 3.$$

(v) The permitivity $\varepsilon = 3 \times 8.85 \times 10^{-12}$

$= 26.5 \times 10^{-12}$ coulomb^2/nt-m^2

(vi) Induced charge on each face of dielectric

$$Q_1 = Q - \frac{Q}{R} = Q\left(\frac{K-1}{K}\right)$$

$$= (1.062 \times 10^{-5})\left(\frac{3-1}{3}\right)$$

$= 7.08 \times 10^{-6}$ coulomb

(vii) Initially, electric intensity between tha plates

$$E = \frac{V}{d} = \frac{6000}{10^{-2}} = 6 \times 10^5 \ \frac{\text{volts}}{\text{metre}}$$

(viii) Final, electric intensity

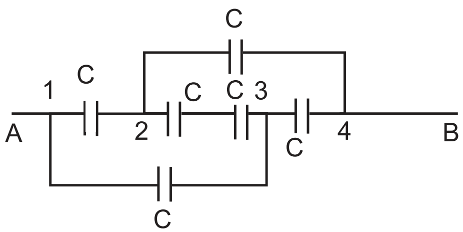
$$E_1 = \frac{2000}{10^{-2}} = 2 \times 10^5 \ \frac{\text{volts}}{\text{metre}}$$

POINT POTENTIAL THEORY

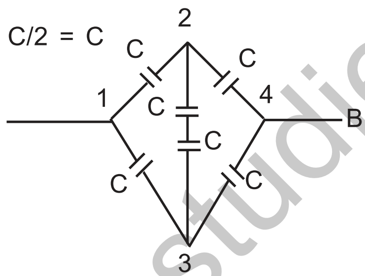
This theory is used to convert a complicated circuit in to a simpler circuit containing only series and, parallel combination of capacitors. We just have to bring out all the points with the same potentials at the same point.

Ex.

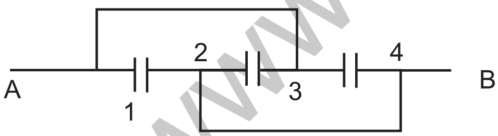
(1)



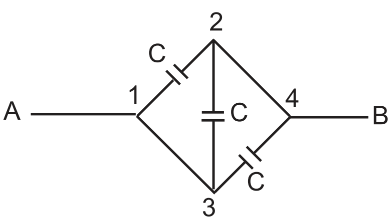
Sol. C_{eq} = C/2 + C/2 = C



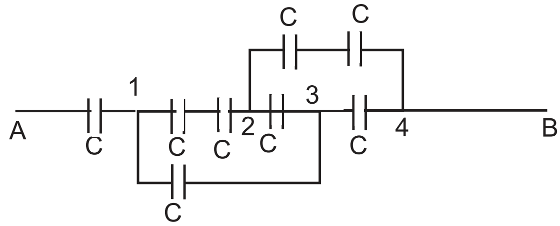
(2)



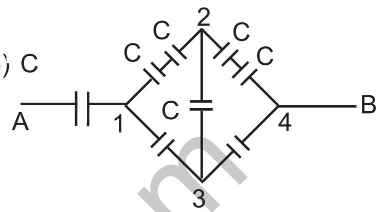
Sol. C_{eq} = C + C + C = 3C



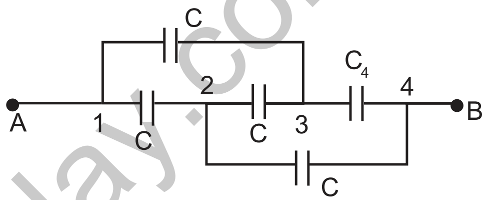
(3)



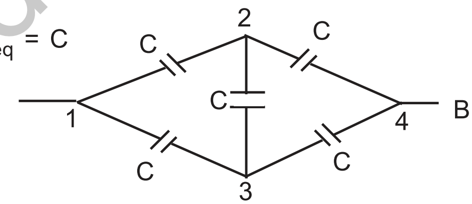
Sol. C_{eq} = (3/5) C



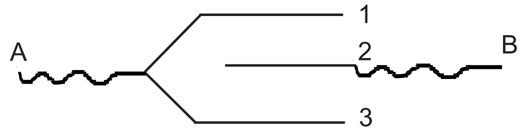
(4)



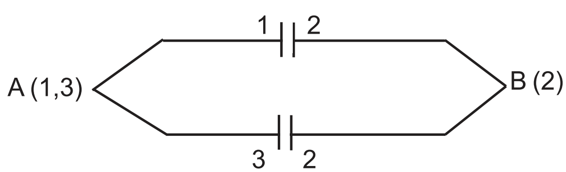
Sol. C_{eq} = C



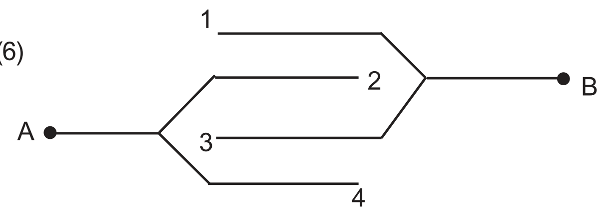
(5)



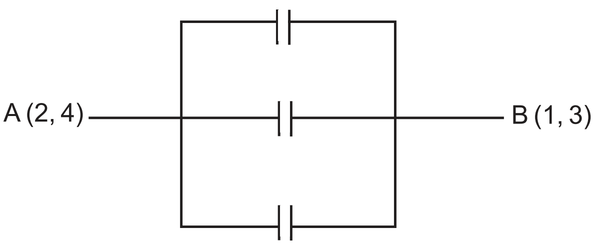
Sol. $C_{eq} = \dfrac{2\varepsilon_0 A}{d}$



(6)



Sol. $C_{eq} = \dfrac{3\varepsilon_0 A}{d}$



If A is connected to positive terminal of battery, then charge on the plates

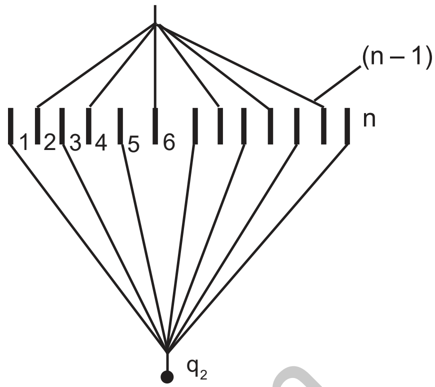
(A) Plate no. 1 = $-\left(\dfrac{\varepsilon_0 A}{d}\right)V$ V

(B) Plate no. 2 = $2\left(\dfrac{\varepsilon_0 A}{d}\right)V$

(C) Plate no. 3 = $-2\left(\dfrac{\varepsilon_0 A}{d}\right)V$

(D) Plate no. 4 = $\left(\dfrac{\varepsilon_0 A}{d}\right)V$

(7)



Sol. $C_{eq} = (n-1)\dfrac{\varepsilon_0 A}{d}$