

Unit - 10

Oscillations And

Waves

SUMMARY

1. **Waves** : The motion of the disturbance in the medium (or in free space) is called wave pulse or generally a wave.
2. **Amplitude of a wave** : Amplitude of oscillation of particles of the medium is called the amplitude of a wave.
3. **Wavelength and frequency** : The linear distance between any two points or particles having phase difference of 2π rad is called the wavelength (λ) of the wave.

Frequency of wave is just the frequency of oscillation of particles of the medium. Relation between wavelength and frequency :

$$v = f\lambda = \frac{\omega}{k} \text{ where, } v \text{ is the speed of wave in the medium.}$$

4. **Mechanical waves** : The waves which require elastic medium for their transmission are called mechanical waves, e.g. sound waves.
5. **Transverse and longitudinal waves** : Waves in which the oscillations are in a direction perpendicular to the direction of wave propagation are called the transverse wave.
Waves in which the oscillations of the particles of medium are along the direction of wave propagation are called longitudinal waves.
6. **Wave Equation** : The equation which describe the displacement for any particle of medium at a required time is called wave equation. Various forms of wave equations are as follows :

$$(i) \ y = A \sin (\omega t - kx)$$

$$(ii) \ y = A \sin \left(\frac{t}{T} - \frac{x}{\lambda} \right)$$

$$(iii) \ y = A \sin 2\pi \left(t - \frac{x}{v} \right)$$

$$(iv) \ y = A \sin \frac{2\pi}{\lambda} (vt - x)$$

The above equations are for the wave travelling in the direction of increasing value of x. If the wave is travelling in the direction of decreasing value of x then put '+' instead of '-' in above equations.

7. The elasticity and inertia of the medium are necessary for the propagation of the mechanical waves.

8. The speed of the transverse waves in a medium like string kept under tension, $v = \sqrt{\frac{T}{\mu}}$

where, T = Tension in the string and (μ = mass per unit length of the string -- y

9. Speed of sound waves in elastic medium, $v = \sqrt{\frac{E}{\rho}}$

where, E = Elastic constant of a medium, ρ = Density of the medium.

$$\text{Speed of longitudinal waves in a fluid, } v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

where, B — Bulk modulus of a medium $\gamma = \frac{C_p}{C_v} = 1.41$ (for air)

Speed of longitudinal waves in a linear medium like a rod, $v = \sqrt{\frac{\gamma}{\rho}}$

where, γ = Young modulus, ρ = Density of a medium

At constant pressure and constant humidity, speed of sound waves in gas is directly proportional to the square root of its absolute temperature.

$$v = \sqrt{\frac{\gamma RT}{M}} \therefore v \propto \sqrt{T}$$

The speed of sound in a gas does not depend on the pressure variation.

10. Principle of Superposition : When a particle of medium comes under the influence of two or more waves simultaneously, its net displacement is the vector sum of displacement that could occur under the influence of the individual waves.
11. Stationary Waves : When two waves having same amplitude and frequency and travelling in mutually opposite directions are superposed the resultant wave formed loses the property of propagation. Such a wave is called a stationary wave.

Equation of stationary wave : $y = -2A \sin kx \cos \omega t$

Amplitude of stationary wave : $2A \sin kx$

Position of nodes in stationary wave $x_n = \frac{n\lambda}{2}$

where, $n = 1, 2, 3, \dots$. At all these points the amplitude is zero.

Position of antinodes in stationary wave's,

$$x_n = (2n - 1) \frac{\lambda}{4} \text{ where } n = 1, 2, 3, \dots$$

The amplitude of all these points is $2A$.

12. Frequencies corresponding to different normal modes of vibration in a stretched string of length L fixed at both the ends are given by,

$$f_n = \frac{nv}{2L} = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \text{ where } n = 1, 2, 3, \dots$$

13. In a closed pipe the values of possible wavelengths required for stationary wave pattern are given by.

$$\lambda_n = \frac{4L}{(2n - 1)} \text{ and possible frequencies, } f_n = (2n - 1) \frac{v}{4L} = (2n - 1) f_1$$

where, $n = 1, 2, 3, \dots$ and L = length of pipe.

In a closed pipe only odd harmonics $f_1, 3f_1, 5f_1, \dots$ are possible.

14. In an open pipe the values of possible wavelength required for stationary waves are given by,

$$\lambda_n = \frac{2L}{n} \text{ and possible frequencies, } f_n = \frac{nv}{2L} = nf_1 \text{ where, } n = 1, 2, 3, \dots \text{ and}$$

L - length of pipe.

In open pipe of the harmonics like $f_1, 2f_1, 3f_1, \dots$ are possible.

15. Beat: The phenomenon of the loudness of sound becoming maximum periodically due to superposition of two sound waves of equal amplitude and slightly different frequencies is called the 'beats'.

$$\text{Number of beats produced in unit time} = f_1 - f_2.$$

16. Doppler Effect : Whenever there is a relative motion between a source of sound and a listener with respect to the medium in which the waves are propagating the frequency of sound experienced by the listener is different from that which is emitted by the source. This phenomenon is called Doppler effect.

$$\text{Frequency listened by the listener, } f_l = \frac{v \pm v_L}{v \pm v_s} f_s$$

Where, v = velocity of sound, v_L = velocity of a listener,

v_s = velocity of a source, f_s = frequency of sound emitted by the source.

17. If a body repeats its motion along a certain path, about a fixed point, at a definite interval of time, it is said to have periodic motion.
18. If a body moves to and fro, back and forth, or up and down about a fixed point in a fixed interval of time, such a motion is called an oscillatory motion.
19. When a body moves to and fro repeatedly about an equilibrium position under a restoring force, which is always directed towards equilibrium position and whose magnitude at any instant is directly proportional to the displacement of the body from the equilibrium position of that instant then such a motion is known as simple harmonic motion.
20. The maximum displacement of the oscillator on either side of mean position is called amplitude of the oscillator.
21. The time taken by the oscillator to complete one oscillation is known as periodic time or time period or period (T) of the oscillator.
22. The number of oscillation completed by the simple harmonic oscillator in one second is known as its frequency (f).
23. 2π times the frequency of oscillator is the angular frequency ω of that oscillator.

$$24. \quad T = \frac{1}{f} = \frac{2\pi}{\omega} \text{ or } f = \frac{1}{T} \text{ or } \omega = \frac{2\pi}{T}$$

25. For simple harmonic motion, the displacement $y(t)$ of a particle from its equilibrium position is represented by sine, cosine or its linear combination like

$$y(t) = A \sin(\omega t + \phi)$$

$$y(t) = B \cos(\omega t + \phi)$$

$$y(t) = A' \sin \omega t + B' \cos \omega t$$

$$\text{where } A' = A \cos \phi \text{ and } B' = B \sin \phi$$

26. The velocity of SHO is given by $v = \pm \omega \sqrt{A^2 - y^2}$
27. The acceleration of SHO is given by $a = -\omega^2 y$
28. A particle of mass m oscillating under the influence of Hook's Law exhibits simple harmonic motion with

$$\omega = \sqrt{\frac{k}{m}};$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

29. Differential equation for SHM is $\frac{d^2 y}{dt^2} + \omega^2 y = 0$
30. For series combination of n spring of spring constants $k_1, k_2, k_3, \dots, k_n$, the equivalent spring constant is

$$\frac{1}{k} = \frac{1}{k_1} = \frac{1}{k_2} + \dots + \frac{1}{k_n} \text{ the periodic time } T = 2\pi\sqrt{\frac{m}{k}}$$

31. For parallel combination of n springs of spring constants k_1, k_2, \dots, k_n , the equivalent spring constant is

$$k = k_1 + k_2 + k_3 + \dots + k_n \text{ and period } T = 2\pi\sqrt{\frac{m}{k}}$$

32. The kinetic energy of the SHO is $K = \frac{1}{2} m \omega^2 (A^2 - y^2)$

33. The potential energy of the SHO is $U = \frac{1}{2} k y^2$

34. The total mechanical energy of SHO is $E = K + U = \frac{1}{2} m \omega^2 A^2 = \frac{1}{2} k A^2$

35. For SHO, at $y = 0$, the potential energy is minimum ($U = 0$) and the kinetic energy is maximum

$$(K = \frac{1}{2} k A^2 = E)$$

36. For SHO, at $y = \pm A$, the potential energy is maximum ($U = \frac{1}{2} k A^2 = E$) and the kinetic energy is minimum ($K = 0$)

37. Simple harmonic motion is the projection of uniform circular motion on a diameter of the reference circle.

38. For simple pendulum, for small angular displacement

$$T = 2\pi \sqrt{\frac{l}{g}} \text{ and}$$

$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

39. For simple pendulum, T is independent of the mass of the bob as well as the amplitude of the oscillations.

40. The differential equation for damped harmonic oscillation is

with the displacement

$$m \frac{d^2 y}{dt^2} + b \frac{dy}{dt} + ky = 0$$

and angular frequency $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

41. $E(t) = \frac{1}{2} kA^2 e^{-ct/m}$ gives the mechanical energy of damped oscillation at time t.

42. A system oscillates under the influence of external periodic force are forced oscillations.

43. The differential equation for forced oscillations is

$$\frac{d^2 y}{dt^2} + \frac{b}{m} \frac{dy}{dt} + \frac{k}{m} y = \frac{F_0}{m} \sin \omega t$$

44. The amplitude for forced oscillation is

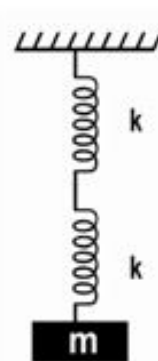
$$A = \frac{F_0}{[m^2(\omega_0 - \omega)^2 + b^2\omega^2]^{\frac{1}{2}}}$$

MCQ

For the answer of the following questions choose the correct alternative from among the given ones.

SECTION – I:

- If the equation for a particle performing S.H.M. is given by $y = \sin 2t + \sqrt{3} \cos 2t$, its periodic time will bes.
 (A) 21 (B) p (C) 2p (D) 4p.
- The distance travelled by a particle performing S.H.M. during time interval equal to its periodic time is
 (A) A (B) 2A (C) 4A (D) Zero.
- A person standing in a stationary lift measures the periodic time of a simple pendulum inside the lift to be equal to T. Now, if the lift moves along the vertically upward direction with an acceleration of $\frac{g}{3}$, then the periodic time of the lift will now be
 (A) $\sqrt{3}T$ (B) $\frac{\sqrt{3}}{2}T$ (C) $\frac{T}{3}$ (D) $\frac{T}{\sqrt{3}}$
- If the equation for displacement of two particles executing S.H.M. is given by $y_1 = 2\sin(10t + \theta)$ and $y_2 = 3\cos 10t$ respectively, then the phase difference between the velocity of two particles will be
 (A) $-\theta$ (B) θ (C) $\theta - \frac{\pi}{2}$ (D) $\theta + \frac{\pi}{2}$.
- When a body having mass m is suspended from the free end of two springs suspended from a rigid support, as shown in figure, its periodic time of oscillation is T. If only one of the two springs are used, then the periodic time would be
 (A) $\frac{T}{\sqrt{2}}$ (B) $\frac{T}{2}$
 (C) $\sqrt{2} T$ (D) 2T
- If the maximum velocity of two springs (both has same mass) executing S.H.M. and having force constants k_1 and k_2 respectively are same, then the ratio of their amplitudes will be
 (A) $\frac{k_1}{k_2}$ (B) $\frac{k_2}{k_1}$ (C) $\sqrt{\frac{k_1}{k_2}}$ (D) $\sqrt{\frac{k_2}{k_1}}$



7. As shown in figure, two masses of 3.0 kg and 1.0 kg are attached at the two ends of a spring having force constant 300 N m^{-1} . The natural frequency of oscillation for the system will be Hz. (Ignore friction)

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$
(C) 4 (D) 3



8. The bob of a simple pendulum having length ' l ' is displaced from its equilibrium position by an angle of θ and released. If the velocity of the bob, while passing through its equilibrium position is v , then $v = \dots\dots\dots$

(A) $\sqrt{2gl(1 - \cos\theta)}$ (B) $\sqrt{2gl(1 + \sin\theta)}$
(C) $\sqrt{2gl(1 - \sin\theta)}$ (D) $\sqrt{2gl(1 + \cos\theta)}$

9. If $\frac{1}{4}$ of a spring having length l is cutoff, then what will be the spring constant of remaining part?

(A) k (B) $4k$ (C) $\frac{4k}{3}$ (D) $\frac{3k}{4}$

10. The amplitude for a S.H.M. given by the equation $x = 3\sin 3\pi t + 4\cos 3\pi t$ is m.

(A) 5 (B) 7 (C) 4 (D) 3.

11. When an elastic spring is given a displacement of 10mm, it gains an potential energy equal to U . If this spring is given an additional displacement of 10 mm, then its potential energy will be

(A) U (B) $2U$ (C) $4U$ (D) $U/4$.

12. The increase in periodic time of a simple pendulum executing S.H.M. iswhen its length is increased by 21%.

(A) 42 % (B) 10% (C) 11% (D) 21%.

13. A particle executing S.H.M. has an amplitude A and periodic time T . The minimum time required by the particle to get displaced by $\frac{A}{\sqrt{2}}$ from its equilibrium position is s.

(A) T (B) $T/4$ (C) $T/8$ (D) $T/16$.

14. If a body having mass M is suspended from the free ends of two springs A and B, their periodic time are found to be T_1 and T_2 respectively. If both these springs are now connected in series and if the same mass is suspended from the free end, then the periodic time is found to be T . Therefore

(A) $T = T_1 + T_2$ (B) $\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$ (C) $T^2 = T_1^2 + T_2^2$ (D) $\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$.

15. The displacement of a S.H.O. is given by the equation $x = A \cos(\omega t + \frac{\pi}{8})$. At what time will it attain maximum velocity?

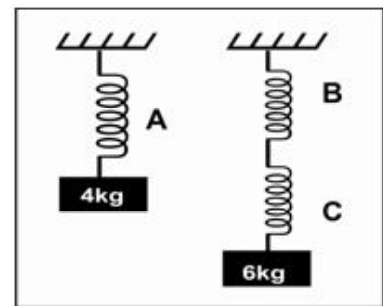
(A) $\frac{3\pi}{8\omega}$ (B) $\frac{8\pi}{3\omega}$ (C) $\frac{3\pi}{16\omega}$ (D) $\frac{\pi}{16\omega}$.

16. At what position will the potential energy of a S.H.O. become equal to one third its kinetic energy?

(A) $\pm \frac{A}{2}$ (B) $\pm \frac{A}{\sqrt{2}}$ (C) $\pm \frac{A}{\sqrt{3}}$ (D) $\pm \sqrt{3}A$.

17. Three identical springs are shown in figure. When a 4 kg mass is suspended from spring A, its length increases by 1 cm. Now if a 6 kg mass is suspended from the free end of spring C, then increase in its length is cm.

(A) 1.5 (B) 3.0
(C) 4.5 (D) 6.0.



18. For particles A and B executing S.H.M., the equation for displacement is given by $y_1 = 0.1 \sin(100t + \frac{\pi}{3})$ and $y_2 = 0.1 \cos pt$ respectively. The phase difference between velocity of particle A with respect to that of B is

(A) $-\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $-\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

19. The periodic time of a simple pendulum is T_1 . Now if the point of suspension of this pendulum starts moving along the vertical direction according to the equation $y = kt^2$, the periodic time

of the pendulum becomes T_2 . Therefore, $\frac{T_1^2}{T_2^2} = \dots$ ($k = 1 \text{ m/s}^2$ & $g = 10 \text{ m/s}^2$)

(A) 6/5 (B) 5/6 (C) 4/5 (D) 1

20. A hollow sphere is filled with water. There is a hole at the bottom of this sphere. This sphere is suspended with a string from a rigid support and given an oscillation. During oscillation, the hole is opened up and the periodic time of this oscillating system is measured. The periodic time of the system.....

- (A) will remain constant
(B) Will increase upto a certain time
(C) Increases initially and then decreases to attain its initial periodic time
(D) Initially decreases and then will attain the initial periodic time value.

21. The periodic time of a S.H.O. oscillating about a fixed point is 2 s. After what time will the kinetic energy of the oscillator become 25% of its total energy?

- (A) 1/12 s (B) 1/6 s (C) 1/4 s (D) 1/3 s.

22. A body having mass 5g is executing S.H.M. with an amplitude of 0.3 m. If the periodic time of the system is $\frac{\pi}{10}$ s, then the maximum force acting on body is

- (A) 0.6 N (B) 0.3 N (C) 6 N (D) 3 N

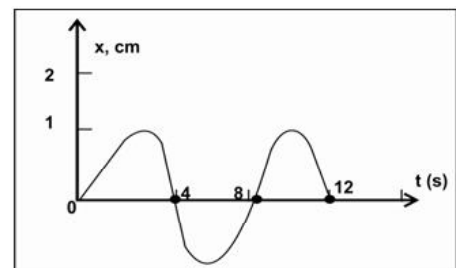
23. As shown in figure, a body having mass m is attached with two springs having spring constants k_1 and k_2 . The frequency of oscillation is f . Now, if the springs constants of both the springs are increased 4 times, then the frequency of oscillation will be equal to

- (A) 2f (B) f/2
(C) f/4 (D) 4f



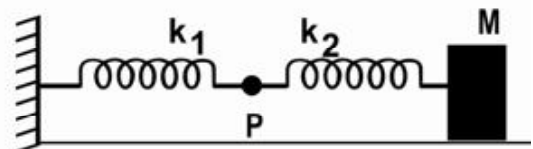
24. The figure shows a graph of displacement versus time for a particle executing S.H.M. The acceleration of the S.H.O. at the end of time $t = \frac{4}{3}$ second iscm.s⁻²

- (A) $\frac{\sqrt{3}}{32} \pi^2$ (B) $-\frac{\pi^2}{32}$
(C) $\frac{\pi^2}{32}$ (D) $-\frac{\sqrt{3}}{32} \pi^2$



25. As shown in figure, the object having mass M is executing S.H.M. with an amplitude A . The amplitude of point P shown in figure will be

- (A) $\frac{k_1 A}{k_2}$ (B) $\frac{k_2 A}{k_1}$
(C) $\frac{k_1 A}{k_1 + k_2}$ (D) $\frac{k_2 A}{k_1 + k_2}$



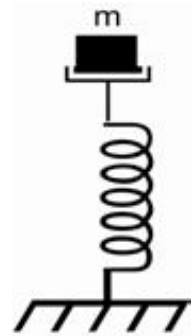
26. A particle is executing S.H.M. between $x = -A$ and $x = +A$. If the time taken by the particle to travel from $x = 0$ to $A/2$ is T_1 and that taken to travel from $x = A/2$ to $x = A$ is T_2 , then

- (A) $T_1 < T_2$ (B) $T_1 > T_2$ (C) $T_1 = 2T_2$ (D) $T_1 = T_2$

27. For a particle executing S.H.M., when the potential energy of the oscillator becomes 1/8 the maximum potential energy, the displacement of the oscillator in terms of amplitude A will be

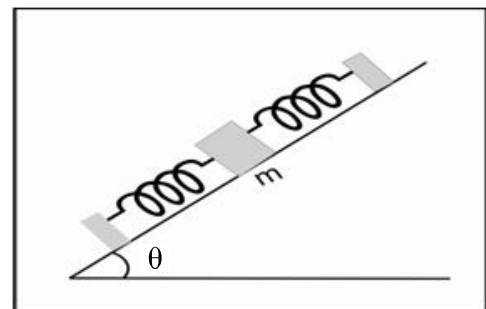
- (A) $\frac{A}{\sqrt{2}}$ (B) $\frac{A}{2\sqrt{2}}$ (C) $\frac{A}{2}$ (D) $\frac{A}{3\sqrt{2}}$.

28. The average values of potential energy and kinetic energy over a cycle for a S.H.O. will be respectively.
- (A) $0, \frac{1}{2}m\omega^2 A^2$ (B) $\frac{1}{2}m\omega^2 A^2, 0$
 (C) $\frac{1}{2}m\omega^2 A^2, \frac{1}{2}m\omega^2 A^2$ (D) $\frac{1}{4}m\omega^2 A^2, \frac{1}{4}m\omega^2 A^2$.
29. The ratio of force constants of two springs is 1:5. The equal mass suspended at the free ends of both springs are performing S.H.M. If the maximum acceleration for both springs are equal, the ratio of amplitudes for both springs is
- (A) $\frac{1}{\sqrt{5}}$ (B) $\frac{1}{5}$ (C) $\frac{5}{1}$ (D) $\frac{\sqrt{5}}{1}$
30. When a mass M is suspended from the free end of a spring, its periodic time is found to be T. Now, if the spring is divided into two equal parts and the same mass M is suspended and oscillated, the periodic time of oscillation is found to be T'. Then
- (A) $T < T'$ (B) $T = T'$ (C) $T > T'$ (D) Nothing can be said.
31. The periodic time of two oscillators are T and $\frac{5T}{4}$ respectively. Both oscillators starts their oscillation simultaneously from the mid point of their path of motion. When the oscillator having periodic time T completes one oscillation, the phase difference between the two oscillators will be
- (A) 90° (B) 112° (C) 72° (D) 45°
32. A rectangular block having mass m and cross sectional area A is floating in a liquid having density ρ . If this block in its equilibrium position is given a small vertical displacement, it starts oscillating with periodic time T. Then in this case.....
- (A) $T \propto \frac{1}{\sqrt{m}}$ (B) $T \propto \sqrt{\rho}$ (C) $T \propto \frac{1}{\sqrt{A}}$ (D) $T \propto \frac{1}{\sqrt{\rho}}$
33. As shown in figure, a spring attached to the ground vertically has a horizontal massless plate with a 2 kg mass in it. When the spring (massless) is pressed slightly and released, the 2 kg mass, starts executing S.H.M. The force constant of the spring is 200 N m^{-1} . For what minimum value of amplitude, will the mass loose contact with the plate? (Take $g = 10 \text{ ms}^{-2}$)
- (A) 10.0 cm (B) 8.0 cm
 (C) 4.0 cm (D) For any value less than 12.0 cm.



34. Which of the equation given below represents a S.H.M.?
 (A) acceleration = $-k(x + a)$ (B) acceleration = $k(x + a)$
 (C) acceleration = kx (D) acceleration = $-k_0x + k_1x^2$
 { Here k , k_0 and k_1 are force constants and units of x and a is meter }
35. The displacement for a particle performing S.H.M. is given by $x = A \cos(\omega t + \hat{O})$. If the initial position of the particle is 1 cm and its initial velocity is $p \text{ cm s}^{-1}$, then what will be its initial phase? The angular frequency of the particle is $p \text{ s}^{-1}$.
 (A) $\frac{2\pi}{4}$ (B) $\frac{7\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{3\pi}{4}$
36. Two simple pendulums having lengths 144 cm and 121 cm starts executing oscillations. At some time, both bobs of the pendulum are at the equilibrium positions and in same phase. After how many oscillations of the shorter pendulum will both the bob's pass through the equilibrium position and will have same phase?
 (A) 11 (B) 12 (C) 21 (D) 20
37. The maximum velocity and maximum acceleration of a particle executing S.H.M. are 1 m/s and 3.14 m/s^2 respectively. The frequency of oscillation for this particle is
 (A) 0.5 s^{-1} (B) 3.14 s^{-1} (C) 0.25 s^{-1} (D) 2 s^{-1}
38. A particle having mass 1 kg is executing S.H.M. with an amplitude of 0.01 m and a frequency of 60 hz. The maximum force acting on this particle is N
 (A) $144p^2$ (B) $288p^2$ (C) $188p^2$ (D) None of these.
39. A simple pendulum having length l is given a small angular displacement at time $t = 0$ and released. After time t , the linear displacement of the bob of the pendulum is given by
 (A) $x = a \sin 2p \sqrt{\frac{l}{g}} t$ (B) $x = a \cos 2p \sqrt{\frac{g}{l}} t$ (C) $x = a \sin \sqrt{\frac{g}{l}} t$ (D) $x = a \cos \sqrt{\frac{g}{l}} t$
40. Two masses m_1 and m_2 are attached to the two ends of a massless spring having force constant k . When the system is in equilibrium, if the mass m_1 is detached, then the angular frequency of mass m_2 will be
 (A) $\sqrt{\frac{k}{m_1}}$ (B) $\sqrt{\frac{k}{m_2}}$ (C) $\sqrt{\frac{k}{m_2} + m_1}$ (D) $\sqrt{\frac{k}{m_1 + m_2}}$
41. When the displacement of a S.H.O. is equal to $A/2$, what fraction of total energy will be equal to kinetic energy? { A is amplitude }
 (A) $2/7$ (B) $3/4$ (C) $2/9$ (D) $5/7$

42. The speed of a particle executing motion changes with time according to the equation $y = a\sin\omega t + b\cos\omega t$, then
- (A) Motion is periodic but not a S.H.M.
 (B) It is a S.H.M. with amplitude equal to $a+b$
 (C) It is a S.H.M. with amplitude equal to $a^2 + b^2$
 (D) Motion is a S.H.M. with amplitude equal to $\sqrt{a^2 + b^2}$.
43. A body is placed on a horizontal plank executing S.H.M. along vertical direction. Its amplitude of oscillation is $3.92 \times 10^{-3}\text{m}$. What should be the minimum periodic time so that the body does not lose contact with the plank?
- (A) 0.1256 s (B) 0.1356 s (C) 0.1456 s (D) 0.1556 s
44. If the kinetic energy of a particle executing S.H.M. is given by $K = K_0 \cos^2\omega t$, then the displacement of the particle is given by
- (A) $\left(\frac{K_0}{m\omega^2}\right)^{1/2} \sin\omega t$ (B) $\left(\frac{2K_0}{m\omega^2}\right)^{1/2} \sin\omega t$ (C) $\left(\frac{2\omega^2}{mK_0}\right)^{1/2} \sin\omega t$ (D) $\left(\frac{2K_0}{m\omega}\right)^{1/2} \sin\omega t$
45. The equation for displacement of two identical particles performing S.H.M. is given by $x_1 = 4\sin(20t + \pi/6)$ and $x_2 = 10\sin\omega t$. For what value of ω will both particles have same energy?
- (A) 4 units (B) 8 units (C) 16 units (D) 20 units
46. A spring having length l and spring constant k is divided into two parts having lengths l_1 and l_2 . If $l_1 = nl_2$, the force constant of the spring having length l_2 is
- (A) $k(1+n)$ (B) $k\left(\frac{1+n}{n}\right)$ (C) k (D) $\frac{k}{(n+1)}$
47. When a mass m is suspended from the free end of a massless spring having force constant k , it oscillates with frequency f . Now if the spring is divided into two equal parts and a mass $2m$ is suspended from the end of any one of them, it will oscillate with a frequency equal to
- (A) f (B) $2f$ (C) $\frac{f}{\sqrt{2}}$ (D) $\sqrt{2}f$
48. A mass m on an inclined smooth surface is attached to two springs as shown in figure. The other ends of both springs are attached to rigid surface. If the force constant of both spring is k , then the periodic time of oscillation for the system is
- (A) $2\pi\left(\frac{M}{2k}\right)^{1/2}$ (B) $2\pi\left(\frac{2M}{k}\right)^{1/2}$
 (C) $2\pi\left(\frac{Mg\sin\theta}{2k}\right)^{1/2}$ (D) $2\pi\left(\frac{2Mg}{k}\right)^{1/2}$



49. A body of mass 1 kg suspended from the free end of a spring having force constant 400 Nm^{-1} is executing S.H.M. When the total energy of the system is 2 joule, the maximum acceleration is ms^{-2} .

- (A) 8 ms^{-2} (B) 10 ms^{-2} (C) 40 ms^{-2} (D) 40 cms^{-2}

50. When a block of mass m is suspended from the free end of a massless spring having force constant k , its length increases by y . Now when the block is slightly pulled downwards and released, it starts executing S.H.M with amplitude A and angular frequency ω . The total energy of the system comprising of the block and spring is

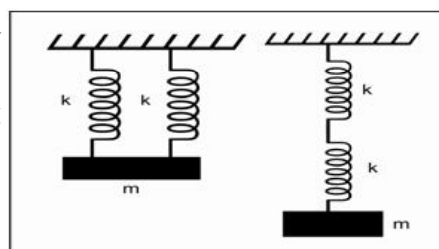
- (A) $\frac{1}{2} m \omega^2 A^2$ (B) $\frac{1}{2} m \omega^2 A^2 + \frac{1}{2} k y^2$ (C) $\frac{1}{2} k y^2$ (D) $\frac{1}{2} m \omega^2 A^2 - \frac{1}{2} k y^2$.

51. A spring is attached to the center of a frictionless horizontal turn table and at the other end a body of mass 2 kg is attached. The length of the spring is 35 cm. Now when the turn table is rotated with an angular speed of 10 rad s^{-1} , the length of the spring becomes 40 cm then the force constant of the spring is N/m.

- (A) 1.2×10^3 (B) 1.6×10^3 (C) 2.2×10^3 (D) 2.6×10^3

52. As shown in figure (a) and (b), a body of mass m is attached at the ends of the spring system. All springs have the same spring constant k . Now when both systems oscillates along vertical direction, the ratio of their periodic time is

- (A) $\frac{1}{4}$ (B) $\frac{1}{2}$
(C) 2 (D) 4



53. A simple pendulum is executing S.H.M. around point O between the end points B and C with a periodic time of 6 s. If the distance between B and C is 20 cm then in what time will the bob move from C to D? Point D is at the mid-point of C and O.

- (A) 1 s (B) 2 s (C) 3 s (D) 4 s

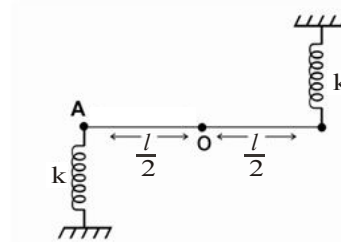
54. A small spherical steel ball is placed at a distance slightly away from the center of a concave mirror having radius of curvature 250 cm. If the ball is released, it will now move on the curved surface. What will be the periodic time of this motion? Ignore frictional force and take $g = 10 \text{ m/s}^2$.

- (A) $\frac{\pi}{4} s$ (B) $p s$ (c) $\frac{\pi}{2} s$ (D) $2p s$

55. Two identical springs are attached at the opposite ends of a rod having length l and mass m . The rod could rotate about its mid-point O as shown in figure. Now, if the point A of the rod is pressed slightly and released, the rod starts executing oscillatory motion. The periodic time of this motion is

- (A) $2\pi \sqrt{\frac{m}{2k}}$ (B) $2\pi \sqrt{\frac{2m}{k}}$

- (C) $\pi \sqrt{\frac{2m}{3k}}$ (D) $\pi \sqrt{\frac{3m}{2k}}$



56. A simple pendulum having length l is suspended at the roof of a train moving with constant acceleration ' a ' along horizontal direction. The periodic time of this pendulum is

(A) $T = 2\pi\sqrt{\frac{l}{g}}$ (B) $T = 2\pi\sqrt{\frac{l}{g+a}}$ (C) $T = 2\pi\sqrt{\frac{l}{g-a}}$ (D) $T = 2\pi\sqrt{\frac{l}{g^2+a^2}}$

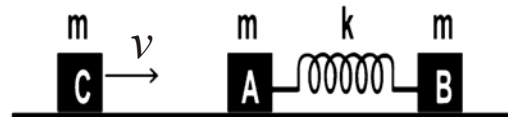
57. A trolley is sliding down a frictionless slope having inclination θ . If a simple pendulum is suspended on top of this trolley, its periodic time is given by $T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}}$, where $g_{\text{eff}} = \dots\dots\dots$

(A) g (B) $g \sin\theta$ (C) $g \cos\theta$ (D) $g \tan\theta$

58. One end of a massless spring having force constant k and length 50 cm is attached at the upper end of a plane inclined at an angle $\theta = 30^\circ$. When a body of mass $m = 1.5$ kg is attached at the lower end of the spring, the length of the spring increases by 2.5 cm. Now, if the mass is displaced by a small amount and released, the amplitude of the resultant oscillation is

(A) $\frac{\pi}{7}$ (B) $\frac{2\pi}{7}$ (C) $\frac{\pi}{5}$ (D) $\frac{2\pi}{5}$

59. Two blocks A and B are attached to the two ends of a spring having length L and force constant k on a horizontal surface. Initially the system is in equilibrium. Now a third block having same mass m , moving with velocity v collides with block A. In this situation.....



- (A) During maximum contraction of the spring, the kinetic energy of the system A-B will be zero.
 (B) During maximum contraction of the spring, the kinetic energy of the system A-B will be $mv^2/4$

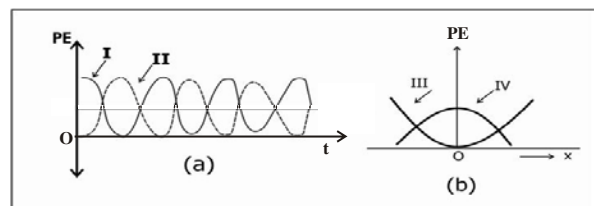
(C) Maximum contraction of the spring is $v\sqrt{\frac{m}{k}}$

(D) Maximum contraction of the spring is $v\sqrt{\frac{2m}{k}}$

60. The displacement of a particle executing S.H.M. is given by $y = 4\cos^2(t/2)\sin 1000t$. This displacement is due to superposition of S.H.M.'s.

(A) 2 (B) 3 (C) 4 (D) 5

61. The displacement of a particle is given by $x = A \cos \omega t$. Which of the following graph represents variation in potential energy as a function of time t and displacement x .



- (A) I, III (B) II, IV
 (C) II, III (D) I, IV

62. A system is executing S.H.M. The potential energy of the system for displacement x is E_1 and for a displacement of y , the potential energy of the system is E_2 . The potential energy for a displacement of $(x+y)$ is

(A) $E_1 + E_2$ (B) $\sqrt{E_1^2 + E_2^2}$ (C) $E_1 + E_2 + 2\sqrt{E_1E_2}$ (D) $\sqrt{E_1E_2}$

63. A system is executing S.H.M. with a periodic time of $4/5$ s under the influence of force F_1 . When a force F_2 is applied, the periodic time is $(2/5)$ s. Now if F_1 and F_2 are applied simultaneously along the same direction, the periodic time will be

(A) $\frac{4}{5\sqrt{5}}$ (B) $\frac{5}{4\sqrt{5}}$ (C) $\frac{8}{4\sqrt{5}}$ (D) $\frac{8}{5\sqrt{5}}$

64. The periodic time of a simple pendulum is 3.3 s. Now if the point of support of the pendulum starts moving along the vertically upward direction with a velocity $v = kt$ (where $k = 2.1 \text{ m/s}^2$), then the new periodic time is s. { Take $g = 10 \text{ m/s}^2$ }

(A) 3 (B) 2.5 (C) 3.33 (D) 2.33

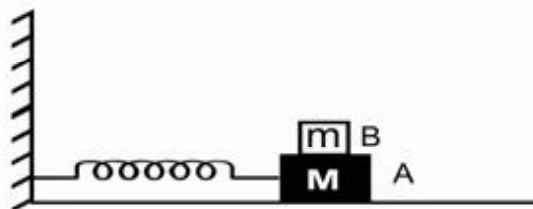
65. A block is placed on a horizontal table. The table executes S.H.M. along the horizontal plane with a period T . The coefficient of static friction between the table and block is μ . The maximum amplitude of oscillation should beso that the block does not slide off the table.

(A) $\frac{\mu g T}{5\pi}$ (B) $\frac{\mu g T^2}{4\pi^2}$ (C) $\frac{\mu g T}{2\pi}$ (D) $\mu g T$

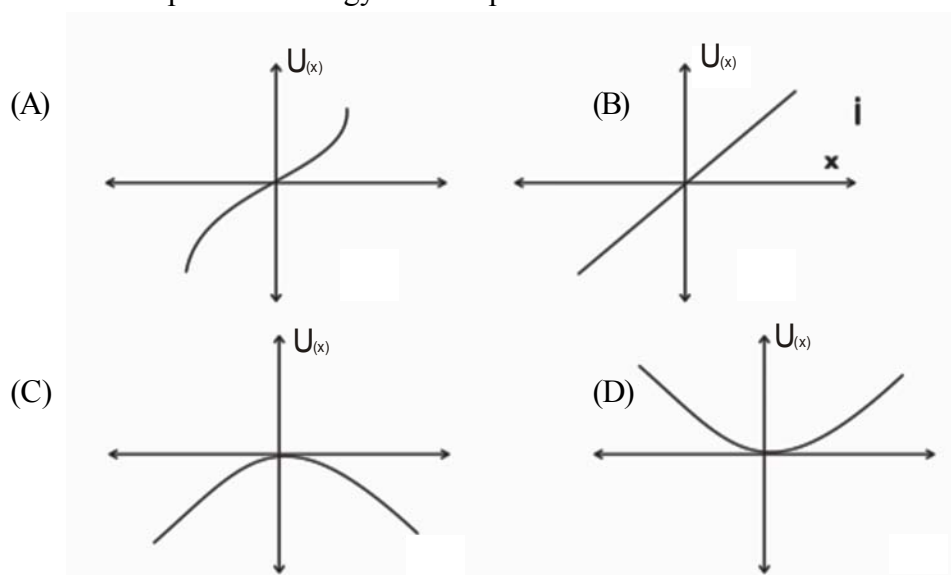
66. As shown in figure, a block A having mass M is attached to one end of a massless spring. The block is on a frictionless horizontal surface and the free end of the spring is attached to a wall. Another block B having mass ' m ' is placed on top of block A. Now on displacing this system horizontally and released, it executes S.H.M. What should be the maximum amplitude of oscillation so that B does not slide off A? Coefficient of static friction between the surfaces of the block's is μ .

(A) $A_{\max} = \frac{\mu mg}{k}$ (B) $A_{\max} = \frac{\mu(m + M)}{k}$

(C) $A_{\max} = \frac{\mu(M - m)g}{k}$ (D) $A_{\max} = \frac{2\mu(M + m)}{k}$



67. A particle is executing S.H.M. about the origin at $x = 0$. Which of the following graph shows variation in potential energy with displacement?



68. A horizontal plank is executing SHM along the vertical direction with angular frequency ω . A coin is placed on top of this plank. If the amplitude of oscillation is increased gradually, for what maximum amplitude will the coin be on the verge of losing contact with the plank?

- (A) When plank is at its maximum height (B) When the plank is at the midpoint.
 (C) When the amplitude is $\frac{g}{\omega^2}$ (D) When the amplitude is $\frac{g^2}{\omega^2}$

SECTION : II

Assertion – Reason type questions :

Note:

For the following questions, statement as well as the reason(s) are given. Each questions has four options. Select the correct option.

- (a) Statement – 1 is true, statement- 2 is true; statement-2 is the correct explanation of statement – 1 .
 (b) Statement – 1 is true, statement- 2 is true but statement-2 is not the correct explanation of statement – 1 .
 (c) Statement – 1 is true, statement- 2 is false
 (d) Statement – 1 is false, statement- 2 is true

- (A) a (B) b (C) c (D) d

69. **Statement – 1 :** If a spring having spring constant k is divided into equal parts, then the spring constant of each part will be $2k$.

Statement – 2 : When the length of the elastic spring is increased (stretched) by x , then the amount of work required to be done is $\frac{1}{2}kx^2$

(A) a (B) b (C) c (D) d

70. **Statement – 1 :** The periodic time of a S.H.O. depends on its amplitude and force constant.

Statement – 2 : The elasticity and inertia decides the frequency of S.H.O.

(A) a (B) b (C) c (D) d

71. **Statement – 1 :** For small amplitude, the motion of a simple pendulum is a S.H.M. with

periodic time $T = 2\pi\sqrt{\frac{l}{g}}$. For large amplitudes, periodic time is greater than $2\pi\sqrt{\frac{l}{g}}$.

Statement – 2 : For large amplitude, the speed of the bob is more when it passes through the mid-point (equilibrium point).

(A) a (B) b (C) c (D) d

72. **Statement – 1 :** Periodic time of a simple pendulum is independent of the mass of the bob.

Statement – 2 : The restoring force does not depend on the mass of the bob.

(A) a (B) b (C) c (D) d

73. **Statement – 1:** The periodic time of a simple pendulum increases on the surface of moon.

Statement – 2 : Moon is very small as compared to Earth.

(A) a (B) b (C) c (D) d

74. **Statement – 1:** If the length of a simple pendulum is increased by 3%, then the periodic time changes by 1.5%.

Statement – 2 : Periodic time of a simple pendulum is proportional to its length.

(A) a (B) b (C) c (D) d

75. **Statement – 1:** For a particle executing S.H.M. with an amplitude of 0.01 m and frequency 30 hz, the maximum acceleration is $36\pi^2 \text{ m/s}^2$.

Statement – 2 : The maximum acceleration for the above particle is $\pm \ddot{u}A$, where A is amplitude.

(A) a (B) b (C) c (D) d

76. **Statement – 1 :** The periodic time of a stiff spring is less than that of a soft spring.

Statement – 2 : The periodic time of a spring depends on its force constant value and for a stiff spring, it is more.

(A) a (B) b (C) c (D) d

77. Statement – 1 : The amplitude of an oscillator decreases with time.

Statement – 2 : The frequency of an oscillator decreases with time.

- (A) a (B) b (C) c (D) d

78. Statement – 1 : For a particle executing SHM, the amplitude and phase is decided by its initial position and initial velocity.

Statement – 2 : In a SHM, the amplitude and phase is dependent on the restoring force.

- (A) a (B) b (C) c (D) d

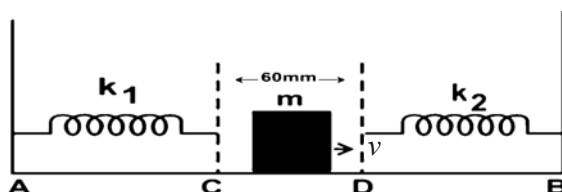
SECTION – III

COMPREHENSION BASED QUESTIONS

NOTE: Questions 79 to 81 are based on the following passage.

Passage – 1:

As shown in figure, two light springs having force constants $k_1 = 1.8 \text{ N m}^{-1}$ and $k_2 = 3.2 \text{ N m}^{-1}$ and a block having mass $m = 200 \text{ g}$ are placed on a frictionless horizontal surface. One end of both springs are attached to rigid supports. The distance between the free ends of the spring is 60 cm and the block is moving in this gap with a speed $v = 120 \text{ cm s}^{-1}$.



79. When the block is moving towards spring k_2 , what will be the time taken for the spring to get maximum compressed from point D ?

- (A) $p \text{ s}$ (B) $(p/2) \text{ s}$ (C) $(p/3) \text{ s}$ (D) $(p/4) \text{ s}$

80. When the block is moving towards k_1 , what will be the time taken for it to get maximum compressed from point C ?

- (A) $p \text{ s}$ (B) $(2/3) \text{ s}$ (C) $(p/3) \text{ s}$ (D) $(p/4) \text{ s}$

81. What will be the periodic time of the block, between the two springs?

- (A) $1 + (5p/6) \text{ s}$ (B) $1 + (7p/6) \text{ s}$ (C) $1 + (5p/12) \text{ s}$ (D) $1 + (7p/12) \text{ s}$

NOTE: Questions 82 to 84 are based on the following passage.

Passage – 2 :

A block having mass M is placed on a horizontal frictionless surface. This mass is attached to one end of a spring having force constant k . The other end of the spring is attached to a rigid wall. This system consisting of spring and mass M is executing SHM with amplitude A and frequency f . When the block is passing through the mid-point of its path of motion, a body of mass m is placed on top of it, as a result of which its amplitude and frequency changes to A' and f' .

82. The ratio of frequencies $\frac{f'}{f} = \dots\dots\dots$

- (A) $\sqrt{\left(\frac{M}{m+M}\right)}$ (B) $\sqrt{\left(\frac{m}{m+M}\right)}$ (C) $\sqrt{\left(\frac{MA}{mA'}\right)}$ (D) $\sqrt{\left(\frac{(M+m)A'}{mA}\right)}$

83. If the velocity before putting the mass and after putting it is v and v' respectively, then $\frac{v'}{v} = \dots\dots\dots$

- (A) $\left(\frac{M}{m+M}\right)$ (B) $\left(\frac{M+m}{M}\right)$ (C) $\left(\frac{M+m}{M-m}\right)\frac{A^1}{A}$ (D) $\left(\frac{M-m}{M+m}\right)\frac{A^1}{A}$

84. The ratio of amplitudes $\frac{A^1}{A} = \dots\dots\dots$

- (A) $\sqrt{\left(\frac{M+m}{m}\right)}$ (B) $\sqrt{\left(\frac{m}{M+m}\right)}$ (C) $\sqrt{\left(\frac{M}{M+m}\right)}$ (D) $\sqrt{\left(\frac{M+m}{M}\right)}$

NOTE: Questions 85 to 90 are based on the following passage.

Passage – 3:

The equation for displacement of a particle at time t is given by the equation $y = 3\cos 2t + 4\sin 2t$.

85. The motion of the particle is

- (A) Damped motion (B) Periodic motion (C) Rotational motion (D) S.H.M.

86. The periodic time of oscillation is

- (A) 2 s (B) p s (C) $(/2)$ s (D) $2p$ s

87. The amplitude of oscillation iscm

- (A) 1 (B) 3 (C) 5 (D) 7

88. The maximum acceleration of the particle iscm / s².

- (A) 4 (B) 12 (C) 20 (D) 28

89. If the mass of the particle is 5 gm, then the total energy of the particle iserg.

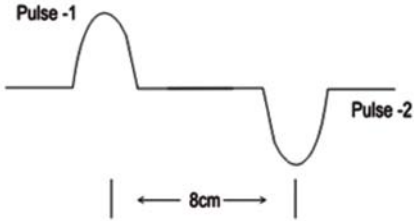
- (A) 250 (B) 125 (C) 500 (D) 375

90. The frequency of the particle iss⁻¹.

- (A) $(1/p)$ (B) p (C) $(1/2p)$ (D) $(p/2)$

Waves**SECTION – I :**

91. Equation for a harmonic progressive wave is given by $y = A \sin (15\pi t + 10\pi x + \pi/3)$ where x is in meter and t is in seconds. This wave is
- (A) Travelling along the positive x direction with a speed of 1.5 ms^{-1} .
 (B) Travelling along the negative x direction with a speed of 1.5 ms^{-1} .
 (C) Has a wavelength of 1.5 m along the $-x$ direction.
 (D) Has a wavelength of 1.5 m along the positive x – direction.
92. If the velocity of sound wave in humid air is v_m and that in dry air is v_d , then.....
- (A) $v_m > v_d$ (B) $v_m < v_d$ (C) $v_m = v_d$ (D) $v_m \gg v_d$
93. The ratio of frequencies of two waves travelling through the same medium is **2:5**. The ratio of their wavelengths will be
- (A) 2:5 (B) 5:2 (C) 3:5 (D) 5:3
94. If the maximum frequency of a sound wave at room temperature is $20,000 \text{ hz}$ then its minimum wavelength will be approximately ($v = 340 \text{ ms}^{-1}$)
- (A) 0.2 A^0 (B) 5 A^0 (C) $5 \text{ cm to } 2 \text{ m}$ (D) 20 mm
95. If the equation of a wave in a string having linear mass density 0.04 kg m^{-1} is given by $y = 0.02 \sin \left[2\pi \left(\frac{t}{0.04} - \frac{x}{0.50} \right) \right]$, then the tension in the string isN. (All values are in mks)
- (A) 6.25 (B) 4.0 (C) 12.5 (D) 0.5
96. If the equation for a transverse wave is $y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$, then for what wavelength will the maximum velocity of the particle be double the wave velocity?
- (A) $\frac{\pi A}{4}$ (B) $\frac{\pi A}{2}$ (C) πA (D) $2\pi A$
97. Consider two points lying at a distance of 10 m and 15 m from an oscillating source. If the periodic time of oscillation is 0.05 s and the velocity of wave produced is 300 m/s , then what will be the phase difference the two points?
- (A) π (B) $\pi/6$ (C) $\pi/3$ (D) $2\pi/3$

98. A string is divided into three parts having lengths l_1 , l_2 and l_3 each. If the fundamental frequency of these parts are f_1 , f_2 and f_3 respectively, then the fundamental frequency of the original string $f = \dots\dots\dots$
- (A) $\sqrt{f} = \sqrt{f_1} + \sqrt{f_2} + \sqrt{f_3}$ (B) $f = f_1 + f_2 + f_3$
- (C) $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$ (D) $\frac{1}{\sqrt{f}} = \frac{1}{\sqrt{f_1}} + \frac{1}{\sqrt{f_2}} + \frac{1}{\sqrt{f_3}}$
99. Waves produced by two tuning forks are given by $y_1 = 4\sin 500\pi t$ and $y_2 = 2\sin 506\pi t$. The number of beats produced per minute is
- (A) 360 (B) 180 (C) 60 (D) 3
100. Equation for a progressive harmonic wave is given by $y = 8\sin 2\pi(0.1x - 2t)$, where x and y are in cm and t is in seconds. What will be the phase difference between two particles of this wave separated by a distance of 2 cm?
- (A) 18° (B) 36° (C) 72° (D) 54°
101. As shown in figure, two pulses in a string having center to center distance of 8 cm are travelling along mutually opposite direction. If the speed of both the pulse is 2 cm/s, then after 2 s, the energy of these pulses will be
- (A) zero
(B) totally kinetic energy
(C) totally potential energy
(D) Partially potential energy and partially kinetic energy.
- 
102. Two waves are represented by $y_1 = A\sin \omega t$ and $y_2 = a\cos \omega t$. The phase of the first wave, w.r.t. to the second wave is
- (A) more by radian (B) less by π radian (C) more by $\pi/2$ (D) less by $\pi/2$
103. If the resultant of two waves having amplitude b is b , then the phase difference between the two waves is
- (A) 120° (B) 60° (C) 90° (D) 180°
104. If two antinodes and three nodes are formed in a distance of 1.21 A° , then the wavelength of the stationary wave is
- (A) 2.42 A° (B) 6.05 A° (C) 3.63 A° (D) 1.21 A°
105. The function $\sin^2(\omega t)$ represents.....
- (A) A SHM with periodic time π/ω . (B) A SHM with a periodic time $2\pi/\omega$.
(C) A periodic motion with periodic time π/ω . (D) A periodic motion with period $2\pi/\omega$.

106. If two almost identical waves having frequencies n_1 and n_2 , produced one after the other superposes then the time interval to obtain a beat of maximum intensity is

(A) $\frac{1}{n_1 - n_2}$ (B) $\frac{1}{n_1} - \frac{1}{n_2}$ (C) $\frac{1}{n_1} + \frac{1}{n_2}$ (D) $\frac{1}{n_1 + n_2}$

107. When two sound waves having amplitude A , angular frequency ω and a phase difference of $\pi/2$ superposes, the maximum amplitude and angular frequency of the resultant wave is

(A) $\sqrt{2}A, \omega$ (B) $\frac{A}{\sqrt{2}}, \frac{\omega}{2}$ (C) $\frac{A}{\sqrt{2}}, \omega$ (D) $\sqrt{2}A, \frac{\omega}{2}$

108. The amplitude of a wave in a string is 2 cm. This wave is propagating along the x-direction with a speed of 128 m/s. Five such waves are accommodated in 4 m length of the string. The equation for this wave is

(A) $y = 0.02\sin(15.7x - 2010t)$ m (B) $y = 0.02\sin(15.7x + 2010t)$ m
(C) $y = 0.02\sin(7.85x - 1005t)$ m (D) $y = 0.02\sin(7.85x + 1005t)$ m

109. A string of length 70 cm is stretched between two rigid supports. The resonant frequency for this string is found to be 420 hz and 315 hz. If there are no resonant frequencies between these two values, then what would be the minimum resonant frequency of this string?

(A) 10.5 hz (B) 1.05 hz (C) 105 hz (D) 1050 hz

110. Sound waves propagates with a speed of 350 m/s through air and with a speed of 3500 m/s through brass. If a sound wave having frequency 700 hz passes from air to brass, then its wavelength

(A) decreases by a fraction of 10 (B) increases 20 times
(C) increases 10 times (D) decreases by a fraction of 20

111. A transverse wave is represented by $y = A\sin(\omega t - kx)$. For what value of its wavelength will the wave velocity be equal to the maximum velocity of the particle taking part in the wave propagation?

(A) $2\pi A$ (B) A (C) πA (D) $\pi A/2$

112. Two monoatomic ideal gases 1 and 2 has molecular weights m_1 and m_2 . Both are kept in two different containers at the same temperature. The ratio of velocity of sound wave in gas 1 and 2 is

(A) $\sqrt{\frac{m_2}{m_1}}$ (B) $\sqrt{\frac{m_1}{m_2}}$ (C) $\frac{m_1}{m_2}$ (D) $\frac{m_2}{m_1}$

113. A wire having length L is kept under tension between $x = 0$ and $x = L$. In one experiment, the equation of the wave and energy is given by $y_1 = A\sin\left(\frac{\pi x}{L}\right)\sin\omega t$ and E_1 respectively. In another

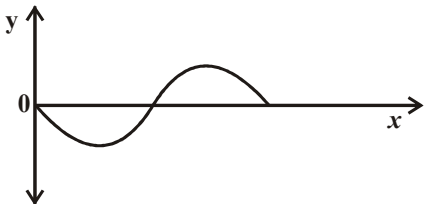
experiment, it is $y_2 = A\sin\left(\frac{2\pi x}{L}\right)\sin 2\omega t$ and E_2 . Then

(A) $E_2 = E_1$ (B) $E_2 = 2E_1$ (C) $E_2 = 4E_1$ (D) $E_2 = 16E_1$

114. Twenty four tuning forks are arranged in such a way that each fork produces 6 beats/s with the preceding fork. If the frequency of the last tuning fork is double than the first fork, then the frequency of the second tuning fork is
- (A) 132 (B) 138 (C) 276 (D) 144
115. If two SHM's are given by the equation $y_1 = 0.1 \sin(100\pi t + \pi/3)$ and $y_2 = 0.1 \cos \pi t$, then the phase difference between the velocity of particle 1 and 2 is
- (A) $\pi/6$ (B) $-\pi/3$ (C) $\pi/3$ (D) $-\pi/6$
116. The wave number for a wave having wavelength 0.005 m is m^{-1} .
- (A) 5 (B) 50 (C) 100 (D) 200
117. An listener is moving towards a stationary source of sound with a speed $\frac{1}{4}$ times the speed of sound. What will be the percentage increase in the frequency of sound heard by the listener?
- (A) 20% (B) 25% (C) 2.5% (D) 5%
118. When the resonance tube experiment, to measure speed of sound is performed in winter, the first harmonic is obtained for 16 cm length of air column. If the same experiment is performed in summer, the second harmonic is obtained for x length of air column. Then
- (A) $32 > x > 16$ (B) $16 > x$ (C) $x > 48$ (D) $48 > x > 32$
119. What should be the speed of a source of sound moving towards a stationary listener, so that the frequency of sound heard by the listener is double the frequency of sound produced by the source? { Speed of sound wave is v }
- (A) v (B) $2v$ (C) $v/2$ (D) $v/4$
120. A metal wire having linear mass density 10 g/m is passed over two supports separated by a distance of 1 m. The wire is kept in tension by suspending a 10 kg mass. The mid point of the wire passes through a magnetic field provided by magnets and an a.c. supply having frequency n is passed through the wire. If the wire starts vibrating with its resonant frequency, what is the frequency of a.c. supply?
- (A) 50 hz (B) 100 hz (C) 200 hz (D) 25 hz
121. If the listener and the source of sound moves along the same direction with the same speed, then.
- (A) $\frac{f_L}{f_s} < 1$ (B) $\frac{f_L}{f_s} = 0$ (C) $\frac{f_L}{f_s} = 1$ (D) $\frac{f_L}{f_s} > 1$
122. A wire of length 10 m and mass 3 kg is suspended from a rigid support. The wire has uniform cross sectional area. Now a block of mass 1 kg is suspended at the free end of the wire and a wave having wavelength 0.05 m is produced at the lower end of the wire. What will be the wavelength of this wave when it reached the upper end of the wire?
- (A) 0.12 m (B) 0.18 m (C) 0.14 m (D) 0.10 m
123. If the mass of 1 mole of air is 29×10^{-3} kg, then the speed of sound in it at STP is ($\gamma = 7/5$). { $T = 273$ K, $P = 1.01 \times 10^5$ Pa }
- (A) 270 m/s (B) 290 m/s (C) 330 m/s (D) 350 m/s

124. A wave travelling along a string is described by $y = 0.005\sin(40x - 2t)$ in SI units. The wavelength and frequency of the wave are.....
- (A) $(\pi/5)$ m ; 0.12 hz (B) $(\pi/10)$ m ; 0.24 hz (C) $(\pi/40)$ m ; 0.48 hz (D) $(\pi/20)$ m ; 0.32 hz
125. Two sitar strings A and B playing the note "Dha" are slightly out of time and produce beats of frequency 5 hz. The tension of the string B is slightly increased and the beat frequency is found to decrease to 3 hz. What is the original frequency of B if the frequency of A is 427 hz?
- (A) 432 (B) 422 (C) 437 (D) 417
126. A rocket is moving at a speed of 130 m/s towards a stationary target. While moving, it emits a wave of frequency 800 hz. Calculate the frequency of the sound as detected by the target. (Speed of wave = 330 m/s)
- (A) 1320 hz (B) 2540 hz (C) 1270 hz (D) 660 hz
127. Length of a steel wire is 11 m and its mass is 2.2 kg. What should be the tension in the wire so that the speed of a transverse wave in it is equal to the speed of sound in dry air at 20° C temperature?
- (A) 2.31×10^4 N (B) 2.25×10^4 N (C) 2.06×10^4 N (D) 2.56×10^4 N
128. A wire stretched between two rigid supports vibrates with a frequency of 45 hz. If the mass of the wire is 3.5×10^{-2} kg and its linear mass density is 4.0×10^{-2} kg/m, what will be the tension in the wire ?
- (A) 212 N (B) 236 N (C) 248 N (D) 254 N
129. Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube A and B is
- (A) 1:2 (B) 1:4 (C) 2:1 (D) 4:1
130. A tuning fork arrangement produces 4 beats/second with one fork of frequency 288 hz. A little wax is applied on the unknown fork and it then produces 2 beats/s. The frequency of the unknown fork ishz.
- (A) 286 (B) 292 (C) 294 (D) 288
131. A wave $y = a\sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is
- (A) $y = a\sin(\omega t + kx)$ (B) $y = -a\sin(\omega t + kx)$ (C) $y = a\sin(\omega t - kx)$ (D) $y = -a\sin(\omega t - kx)$
132. When temperature increases, the frequency of a tuning fork.....
- (A) Increases (B) Decreases
(C) remains same (D) Increases or decreases depending on the material.
133. A tuning fork of known frequency 256 hz makes 5 beats per second with the vibrating string of a piano. The beats frequency decreases to 2 beats/s when the tension in the piano string is slightly increased. The frequency of the piano string before increase in the tension washz.
- (A) $256 + 2$ (B) $256 - 2$ (C) $256 - 5$ (D) $256 + 5$.

134. An observer moves towards a stationary source of sound with a velocity one – fifth the velocity of sound. What is the percentage increase in the apparent frequency?
(A) 5% (B) 20% (C) zero (D) 0.5%
135. The speed of sound in Oxygen (O_2) at a certain temperature is 460 m/s. The speed of sound in helium at the same temperature will bems⁻¹. (Assume both gases to be ideal)
(A) 330 (B) 460 (C) 5002 (D) None of these
136. In a longitudinal wave, pressure variation and displacement variation are.....
(A) In phase (B) 90° out of phase (C) 45° out of phase (D) 180° out of phase
137. A tuning fork of frequency 480 hz produces 10 beats/s when sounded with a vibrating sonometer string. What must have been the frequency of the string if a slight increase in tension produces fewer beats per second than before?
(A) 480 (B) 490 hz (C) 460 hz (D) 470 hz
138. Which of the following functions represents a wave?
(A) $(x - vt)^2$ (B) $\ln(x + vt)$ (C) $e^{-(x+vt)^2}$ (D) $\frac{1}{x + vt}$
139. Two sound waves are represented by $y = a\sin(\omega t - kx)$ and $y = a\cos(\omega t - kx)$. The phase difference between the waves in water is
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) π (D) $\frac{3\pi}{4}$
140. A string of linear density 0.2 kg/m is stretched with a force of 500 N. A transverse wave of length 4.0 m and amplitude 1/l meter is travelling along the string. The speed of the wave is m/s.
(A) 50 (B) 62.5 (C) 2500 (D) 12.5
141. Two wires made up of same material are of equal lengths but their radii are in the ratio 1:2. On stretching each of these two strings by the same tension, the ratio between their fundamental frequency is
(A) 1:2 (B) 2:1 (C) 1:4 (D) 4:1
142. The tension in a wire is decreased by 19%, then the percentage decrease in frequency will be
(A) 19% (B) 10% (C) 0.19% (D) None of these
143. An open organ pipe has fundamental frequency 100 hz. What frequency will be produced if its one end is closed?
(A) 100, 200, 300, (B) 50, 150, 250....
(C) 50, 100, 200, 300... (D) 50, 100, 150, 200,.....
144. A closed organ pipe has fundamental frequency 100 hz. What frequencies will be produced if its other end is also opened?
(A) 200, 400, 600, 800,..... (B) 200, 300, 400, 500,
(C) 100, 300, 500, 700,..... (D) 100, 200, 300, 400,

145. A column of air of length 50 cm resonates with a stretched string of length 40 cm. The length of the same air column which will resonate with 60 cm of the same string at the same tension is
- (A) 100 cm (B) 75 cm (C) 50 cm (D) 25 cm
146. Two forks A and B when sounded together produce 4 beats/s. The fork A is in unison with 30 cm length of a sonometer wire and B is in unison with 25 cm length of the same wire at the same tension. The frequencies of the fork are
- (A) 24 hz, 28 hz (B) 20 hz, 24 hz (C) 16 hz, 20 hz (D) 26 hz, 30 hz
147. A tuning fork of frequency 200 hz is in unison with a sonometer wire. The number of beats heard per second when the tension is increased by 1 % is
- (A) 1 (B) 2 (C) 4 (D) 0.5
148. A bus is moving with a velocity of 5 m/s towards a huge wall. The driver sounds a horn of frequency 165 hz. If the speed of sound in air is 335 m/s, the number of beats heard per second by the passengers in the bus will be
- (A) 3 (B) 4 (C) 5 (D) 6
149. A vehicle with a horn of frequency n is moving with a velocity of 30 m/s in a direction perpendicular to the straight line joining the observer and the vehicle. The observer perceives the sound to have a frequency $(n + n_1)$. If the sound velocity in air is 300 m/s, then.....
- (A) $n_1 = 10n$ (B) $n_1 = 0$ (C) $n_1 = 0.1 n$ (D) $n_1 = - 0.1n$
150. In a sine wave, position of different particles at time $t = 0$ is shown in figure. The equation for this wave travelling along the positive x – direction can be
- (A) $y = A \sin (\omega t - kx)$
 (B) $y = A \cos (kx - \omega t)$
 (C) $y = A \cos (\omega t - kx)$
 (D) $y = A \sin (kx - \omega t)$
- 
151. Which of the following changes at an antinode in a stationary wave?
- (A) Density only (B) Pressure only
 (C) Both pressure and density (D) Neither pressure nor density
152. A sonometer wire supports a 4 kg load and vibrates in fundamental mode with a tuning fork of frequency 416 hz. The length of the wire between the bridges is now doubled. In order to maintain fundamental mode, the load should be changed to
- (A) 1 kg (B) 2 kg (C) 8 kg (D) 16 kg
153. In brass, the velocity of a longitudinal wave is 100 times the velocity of a transverse wave. If $Y = 1 \times 10^{11} \text{ N/m}^2$, then stress in the wire is
- (A) $1 \times 10^{13} \text{ N/m}^2$ (B) $1 \times 10^9 \text{ N/m}^2$ (C) $1 \times 10^{11} \text{ N/m}^2$ (D) $1 \times 10^7 \text{ N/m}^2$.

154. The frequency of tuning fork A is 2% more than the frequency of a standard fork. Frequency of tuning fork B is 3% less than the frequency of the standard fork. If 6 beats per second are heard when the two forks A and B are excited, then frequency of A is hz.
 (A) 120 (B) 122.4 (C) 116.4 (D) 130
155. Fundamental frequency of a sonometer wire is n . If the length and diameter of the wire are doubled keeping the tension same, the new fundamental frequency is
 (A) $\frac{2n}{\sqrt{2}}$ (B) $\frac{n}{2\sqrt{2}}$ (C) $\sqrt{2}n$ (D) $\frac{n}{4}$
156. A car blowing its horn at 480 hz moves towards a high wall at a speed of 20 m/s. If the speed of sound is 340 m/s, the frequency of the reflected sound heard by the driver sitting in the car will be closest to hz.
 (A) 540 (B) 524 (C) 568 (D) 480
157. A cylindrical tube open at both ends has a fundamental frequency f in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now.....
 (A) $f/2$ (B) f (C) $3f/4$ (D) $2f$
158. Three sound waves of equal amplitudes have frequencies $(\nu-1)$, ν , $(\nu+1)$. They superpose to give beats. The number of beats produced per second will be
 (A) 3 (B) 2 (C) 1 (D) 4
159. A wave travelling along the x-axis is described by the equation $y(x,t) = 0.005\cos(\hat{a}x - \hat{a}t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s respectively, then \hat{a} and \hat{a} in appropriate units are
 (A) $\hat{a} = 12.50p$, $\hat{a} = p/2.0$ (B) $\hat{a} = 25p$, $\hat{a} = p$
 (C) $\hat{a} = 0.08/p$, $\hat{a} = 2.0/p$ (D) $\hat{a} = 0.04/p$, $\hat{a} = 1.0/p$
160. A wave travelling along a string is described by the equation $y = A\sin(\hat{u}t - kx)$. The maximum particle velocity is
 (A) $A\hat{u}$ (B) \hat{u}/k (C) $d\hat{u}/dk$ (D) x
161. A string is stretched between fixed points separated by 75 cm. It is observed to have a resonant frequencies of 420 hz and 315 hz. There are other resonant frequencies between these two. Then the lowest frequency for this string is hz.
 (A) 1.05 (B) 1050 (C) 10.5 (D) 105
162. Two tuning forks P and Q when set vibrating gives 4 beats/second. If the prong of fork P is filed, the beats are reduced to 2/s. What is the frequency of P, if that of Q is 250 hz.?
 (A) 246 hz (B) 250 hz (C) 254 hz (D) 252 hz
163. The length of a string tied across two rigid supports is 40 cm. The maximum wavelength of a stationary wave that can be produced in it is cm.
 (A) 20 (B) 40 (C) 80 (D) 120

164. A stationary wave of frequency 200 hz are formed in air. If the velocity of the wave is 360 m/s, the shortest distance between two antinodes ism
 (A) 1.8 (B) 3.6 (C) 0.9 (D) 0.45
165. A tuning fork produces 8 beats per second with both 80 cm and 70 cm of stretched wire of a sonometer. Frequency of the fork ishz.
 (A) 120 (B) 128 (C) 112 (D) 240
166. An open pipe is in resonance in 2nd harmonic with frequency f_1 . Now one end of the tube is closed and frequency is increased to f_2 , such that the resonance again occurs in the nth harmonic. Choose the correct option.
 (A) $n = 3, f_2 = (3/4)f_1$ (B) $n = 3, f_2 = (5/4)f_1$ (C) $n = 5, f_2 = (5/4)f_1$ (D) $n = 5, f_2 = (3/4)f_1$

SECTION : II

Assertion – Reason type questions :

Note:

For the following questions, statement as well as the reason(s) are given. Each questions has four options. Select the correct option.

- (a) Statement – 1 is true, statement- 2 is true; statement-2 is the correct explanation of statement – 1 .
 (b) Statement – 1 is true, statement- 2 is true but statement-2 is not the correct explanation of statement – 1 .
 (c) Statement – 1 is true, statement- 2 is false
 (d) Statement – 1 is false, statement- 2 is true

167. Statement – 1: Two waves moving in a uniform string having uniform tension cannot have different velocities.

Statement – 2 : Elastic and inertial properties of string are same for all waves in same string. Moreover speed of wave in a string depends on its elastic and inertial properties only.

- (A) a (B) b (C) c (D) d

168. Statement – 1: When a sound source moves towards observer, then frequency of sound increases.

Statement – 2 : Wavelength of sound in a medium moving towards the observer decreases.

- (A) a (B) b (C) c (D) d

169. Statement – 1: Newton's equation for speed of sound was found wrong because he assumed the process to be isothermal.

Statement – 2 : When sound propagates, the compressions and rarefactions happen so rapidly that there is not enough time for heat to be distributed.

- (A) a (B) b (C) c (D) d

170. **Statement – 1** : When pressure in a gas changes, velocity of sound in gas may change.

Statement – 2 : Velocity of sound is directly proportional to square root of pressure.

- (A) a (B) b (C) c (D) d

171. **Statement – 1** : If wave enters from one medium to another medium then sum of amplitudes of reflected wave and transmitted wave is equal to the amplitude of incident wave.

Statement – 2 : If wave enters from one medium to another medium some part of energy is transmitted and rest of the energy is reflected back.

- (A) a (B) b (C) c (D) d

SECTION – III

COMPREHENSION BASED QUESTIONS

NOTE: Questions 172 to 174 are based on the following passage.

Passage – 1

A string 25 cm long and having a mass of 2.5 g is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second is heard. It is observed that decreasing the tension in the string decreases the beat frequency. The speed of sound in air is 320 ms^{-1} .

172. The frequency of the fundamental mode of the closed pipe is hz

- (A) 100 (B) 200 (C) 300 (D) 400

173. The frequency of the string vibrating in its 1st overtone is hz

- (A) 92 (B) 108 (C) 192 (D) 208.

174. The tension in the string is very nearly equal to

- (A) 25 N (B) 27 N (C) 28 N (D) 30 N

NOTE: Questions 175 to 178 are based on the following passage.

Passage – 2

Standing waves are produced by the superposition of two waves $y_1 = 0.05\sin(3\pi t - 2x)$ and $y_2 = 0.05\sin(3\pi t + 2x)$ where x and y are in meters and t is in seconds.

175. The speed (in ms^{-1}) of each wave is

- (A) 1.5 (B) 3.0 (C) $3\pi/2$ (D) 3π

176. The distance (in meters) between two consecutive nodes is

- (A) $\pi/2$ (B) π (C) 0.5 (D) 1.0

177. The amplitude of a particle at $x = 0.5$ m is
- (A) 1.08×10^{-1} m (B) 5.4×10^{-2} m (C) $(p/2) \times 10^{-1}$ m (D) $p \times 10^{-1}$ m
178. The velocity (in ms^{-1}) of a particle at $x = 0.25$ m at $t = 0.5$ s is
- (A) $0.1p$ (B) $0.3p$ (C) zero (D) 0.3

NOTE: Questions 179 to 181 are based on the following passage.

Passage – 3

When two sound waves travel in the same direction in a medium, the displacement of a particle located at x at time t is given by $y_1 = 0.05 \cos(0.50\pi x - 100\pi t)$ & $y_2 = 0.05 \cos(0.46\pi x - 92\pi t)$, where y_1, y_2 and x are in meter and t is in seconds.

179. What is the speed of sound in the medium?
- (A) 332 m/s (B) 100 m/s (C) 92 m/s (D) 200 m/s
180. How many times per second does an observer hear the sound of maximum intensity?
- (A) 4 (B) 8 (C) 12 (D) 16
181. At $x = 0$, how many times between $t = 0$ and $t = 1$ s does the resultant displacement become zero?
- (A) 46 (B) 50 (C) 92 (D) 100

NOTE: Questions 182 to 183 are based on the following passage.

Passage – 4

The equation $y = 10 \sin \frac{\pi x}{4} \cos 10t$ represents a stationary wave where x and y are in centimeter and t is in seconds.

182. The amplitude of each component wave is
- (A) 5 cm (B) 10 cm (C) 20 cm (D) between 5 cm and 10 cm.
183. The separation between two consecutive nodes is
- (A) 2 cm (B) 4 cm (C) 5 cm (D) 8 cm

KEY NOTE

1	(B)	41	(B)	81	(D)	121	(C)	161	(D)
2	(C)	42	(D)	82	(A)	122	(D)	162	(A)
3	(B)	43	(A)	83	(A)	123	(C)	163	(C)
4	(C)	44	(B)	84	(C)	124	(D)	164	(C)
5	(A)	45	(B)	85	(D)	125	(B)	165	(A)
6	(D)	46	(A)	86	(B)	126	(A)	166	(C)
7	(D)	47	(A)	87	(C)	127	(A)	167	(D)
8	(A)	48	(A)	88	(C)	128	(C)	168	(A)
9	(C)	49	(C)	89	(A)	129	(C)	169	(A)
10	(A)	50	(B)	90	(A)	130	(B)	170	(B)
11	(C)	51	(B)	91	(B)	131	(B)	171	(D)
12	(B)	52	(B)	92	(A)	132	(B)	172	(B)
13	(C)	53	(A)	93	(B)	133	(C)	173	(D)
14	(C)	54	(B)	94	(D)	134	(B)	174	(B)
15	(A)	55	(C)	95	(A)	135	(D)	175	(C)
16	(A)	56	(D)	96	(C)	136	(D)	176	(A)
17	(B)	57	(C)	97	(D)	137	(D)	177	(B)
18	(C)	58	(A)	98	(C)	138	(C)	178	(C)
19	(A)	59	(B)	99	(B)	139	(A)	179	(D)
20	(C)	60	(B)	100	(C)	140	(A)	180	(A)
21	(D)	61	(A)	101	(B)	141	(B)	181	(D)
22	(A)	62	(C)	102	(D)	142	(B)	182	(A)
23	(A)	63	(A)	103	(A)	143	(B)	183	(B)
24	(D)	64	(A)	104	(D)	144	(A)		
25	(D)	65	(B)	105	(A)	145	(B)		
26	(A)	66	(B)	106	(A)	146	(B)		
27	(B)	67	(D)	107	(A)	147	(A)		
28	(D)	68	(C)	108	(C)	148	(C)		
29	(C)	69	(B)	109	(C)	149	(B)		
30	(C)	70	(D)	110	(C)	150	(D)		
31	(C)	71	(B)	111	(A)	151	(D)		
32	(C)	72	(C)	112	(A)	152	(D)		
33	(A)	73	(B)	113	(C)	153	(D)		
34	(A)	74	(C)	114	(B)	154	(A)		
35	(D)	75	(B)	115	(B)	155	(D)		
36	(B)	76	(A)	116	(D)	156	(A)		
37	(A)	77	(C)	117	(B)	157	(B)		
38	(A)	78	(C)	118	(C)	158	(C)		
39	(D)	79	(D)	119	(C)	159	(B)		
40	(B)	80	(C)	120	(A)	160	(A)		

HINT

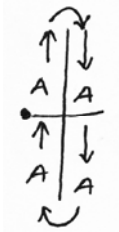
1. $y = \sin 2t + \sqrt{3} \cos 2t$

$$\therefore y = 2 \left\{ \frac{1}{2} \sin 2t + \frac{\sqrt{3}}{2} \cos 2t \right\}$$

$$= 2 \{ \cos \phi \sin 2t + \sin \phi \cos 2t \} = 2 \sin(2t + \phi)$$

Comparing than with $y = A \sin(\omega t + \phi)$, we get

$$\omega = 2 \Rightarrow \frac{2\pi}{T} = 2 \Rightarrow T = \pi \text{ s}$$



3. $T = 2\pi \sqrt{\frac{\ell}{g}}$ When lift moves up with acceleration $\frac{g}{3}$ the effective gravitational accelerations

$$\text{in } g' = g + \frac{g}{3} = \frac{4g}{3}$$

$$\therefore \text{new periodic time } T' = 2\pi \sqrt{\frac{\ell}{g'}}$$

4. $v_1 = \frac{dy_1}{dt} = 2 \times 10 \cos(10t + \theta)$

$$v_2 = -3 \times 10 \sin t = 30 \cos(10t + \frac{\pi}{2})$$

$$\therefore \text{Phase difference} = (10t + \theta) - (10t + \frac{\pi}{2}) = \theta - \frac{\pi}{2}$$

5. For series combination, $K_s = \frac{K_1 k_2}{K_1 + k_2} = \frac{k}{2}$ ($\because k_1 = k_2 = K$)

$$\text{now } T \propto \frac{1}{\sqrt{k}} \therefore \frac{T'}{T} = \sqrt{\frac{k_s}{k}} = \sqrt{2} \Rightarrow T' = \sqrt{2} T$$

6. For maximum velocity, $A_1 \omega_1 = A_2 \omega_2$

$$\therefore \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \sqrt{\frac{k_2/m_2}{k_1/m_1}} = \sqrt{\frac{k_2}{k_1}} \quad (\because m_1 = m_2)$$

7. Reduced mass of system $\mu = \frac{m_1 m_2}{m_1 + m_2} = 0.75 \text{ kg}$

$$\therefore \text{freq of oscillation } f = \frac{1}{2\pi} \sqrt{\frac{k}{\mu}} = \frac{20}{\pi} \approx 3\text{Hz}$$

8.



KE at o = PE at A

$$\therefore \frac{1}{2}mv^2 = mgh = mg\ell\{1 - \cos\theta\}$$

$$v = \sqrt{2g\ell(1 - \cos\theta)}$$

9. $K_1 \ell_1 = K_2 \ell_2 = k\ell$

$$\therefore K_1 \left(\frac{\ell}{4}\right) = K_2 \left(\frac{3}{4}\ell\right) = k\ell$$

\therefore force constant of spring having length $\frac{3}{4} \ell$ in

$$k_2 = \frac{4}{3}k$$

10. Amplitude of SHM given by $x = a \sin \omega t + b \cos \omega t$ in

$$A = \sqrt{a^2 + b^2} = (3^2 + 4^2)^{\frac{1}{2}} = 5\text{m}$$

11. $u \propto y^2 \therefore \frac{u_2}{u_1} = \left(\frac{y_2}{y_1}\right)^2 \Rightarrow u_2 = 4u$

12. $T \propto \sqrt{l}$ because 2π and g are constants

$$\therefore \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{1.21l_1}{l_1}} = 1.1$$

$$\therefore \% \text{increase} = \frac{T_2 - T_1}{T_1} \times 100 = 10\%$$

13. $y = A \sin(\omega t + \phi)$

$$\frac{A}{\sqrt{2}} = A \sin \omega t \{\phi = 0\} \Rightarrow \frac{1}{\sqrt{2}} = \sin \omega t = \frac{\pi}{4}$$

$$\therefore \frac{2\pi}{T} \cdot t = \frac{\pi}{4} \therefore t = \frac{T}{8}$$

$$14. T_1 = 2\pi\sqrt{\frac{m^1}{k_1}} \Rightarrow k_1 = \frac{4\pi^2 \cdot M}{T_1^2} \text{ and } k_2 = \frac{4\pi^2 M}{T_2^2}$$

for Series connection; $T = 2\pi\sqrt{\frac{M}{k}}$ where $k = \frac{k_1 k_2}{k_1 + k_2}$

$$\therefore T = 2\pi\sqrt{\frac{M^1}{4\pi^2 M} (T_1^2 + T_2^2)}$$

$$\therefore T = \sqrt{T_1^2 + T_2^2} \Rightarrow T^2 = T_1^2 + T_2^2$$

$$15. x = A \cos(\omega t + \pi/8) \Rightarrow v = \frac{dx}{dt} = -A\omega \sin(\omega t + \pi/2)$$

If $\sin(\omega t + \pi/8) = 1$, then velocity will be maximum

$$\Rightarrow \omega t + \frac{\pi}{8} = \frac{\pi}{2} \Rightarrow \omega t = \frac{3\pi}{8} \Rightarrow t = \frac{3\pi}{8\omega}$$

$$16. 3u = k$$

$$\therefore 3 \times \frac{1}{2} ky^2 = \frac{1}{2} k(A^2 - y^2) \Rightarrow y = \pm \frac{A}{2}$$

$$17. \text{ For spring } A_b \text{ resting face } F=kx$$

$$\therefore \text{ displacement } x = \frac{F}{k}$$

figure (b) if the resultant spring constant

$$m \circ k^1, \text{ then } k^1 = \frac{k_1 k_2}{k_1 + k_2} \text{ (Series)}$$

$$= \frac{k}{2}$$

\therefore in figure (b) if on applying a force F' , if

$$\text{displacement in } x', \text{ then } x' = \frac{F'}{k'}$$

$$\therefore \frac{x'}{x} = \frac{F'}{k'} \cdot \frac{k}{F} = \frac{6}{4} \times 2 = 3$$

$$\therefore x' = 3x = 3 \times 1\text{cm} = 3\text{cm}$$

$$18. v_1 = \frac{dy_1}{dt}$$

$$\text{and } v_2 = \frac{dy_2}{dt}$$

19. Here $y = kt^2$

$$\therefore \frac{dy}{dt} = 2kt \Rightarrow \frac{d^2y}{dt^2} = 2k = 2 \text{ ms}^{-2}$$

\therefore the point of support is moving upwards with an acceleration of $2 \frac{\text{m}}{\text{s}^2}$

\therefore effective accelⁿ. $g' = g + a = 12 \text{ m/s}^2$

$$\text{Now } T_1 = 2\pi\sqrt{\frac{\ell}{g}} \text{ and } T_2 = 2\pi\sqrt{\frac{\ell}{g'}}$$

20. $T = 2\pi\sqrt{\frac{\ell'}{g}}$ as water leaks, the center of gravity moves down and hence “ ℓ ” increases.

\therefore T increases initially

When all the water has leaked, the center of gravity moves up and hence “ ℓ ” decreases and hence T decreases. Finally the centre of gravity is steady at the center of sphere and so T will remain constant.

21. Kinetic energy = 25 % E

$$\therefore K = \frac{1}{4} E$$

22. $F_{\max} = ma_{\max} = mA\omega^2 = mA \frac{4\pi^2}{T^2} = 0.6 \text{ N}$.

23. For parallel combination $k_p = k_1 + k_2$

$$\therefore \text{frequency } f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}} \rightarrow (1)$$

Now when k_1 and k_2 are increased 4 times,

$$f^1 = \frac{1}{2\pi} \sqrt{\frac{4(K_1 + K_2)}{m}} = 2f. \text{ (from (1))}$$

24. From graph $A=1 \text{ cm} \rightarrow T=8\text{s}$

$$\therefore y = A \sin \omega t = A \sin \frac{2\pi}{T} t \Rightarrow y = \frac{\sqrt{3}}{2} \text{ cm}$$

$$a = -\omega^2 y = \frac{-4\pi^2}{T^2} \cdot \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{3^2} \pi^2 \frac{\text{cm}}{\text{s}^2}$$

25. Now $k \propto \frac{1}{l} \Rightarrow kl = \text{constant}$

$$\Rightarrow k_1 l_1 = k_2 l_2 \Rightarrow l_2 = k_1 l_1$$

$$\text{Now, } A = l_1 \left(\frac{k_1 + k_2}{k_2} \right) \Rightarrow l_1 = \left(\frac{k_2}{k_1 + k_2} \right) A$$

26. $v = w \sqrt{A^2 - x^2}$,

the velocity for moving from $x=0$ to $x = \frac{A}{2}$ will be more than for $x = \frac{A}{2}$ to $x = A$

$$\therefore T_1 < T_2$$

27. $U = \frac{1}{8} u_{\text{max}}$

$$\therefore \frac{1}{2} ky^2 = \frac{1}{8} \left(\frac{1}{2} k A^2 \right) \Rightarrow y^2 = \frac{A^2}{8}$$

28. In the expression for both Kinetic and potential energy, We have the square of the harmonic functions (sine or cosine).

The average of which over a cycle is $\frac{1}{2}$

$$\therefore \langle u \rangle = \frac{E}{2} = \langle K \rangle = \frac{1}{4} m \omega^2 A^2$$

29. Angular frequency $\omega = \sqrt{\frac{k}{m}}$

Since 'm' is constant, $\omega \propto \sqrt{k}^{-1}$

$$\text{Now, } a_{\text{max}} = A \omega^2 \Rightarrow \omega = \sqrt{\frac{a_{\text{max}}}{A}}$$

$$\therefore \frac{a_{\text{max}}}{A} = k \Rightarrow \frac{a_{\text{max}}}{K} = A$$

$$\therefore A \propto \frac{1}{k}$$

30. For a spring, $T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow T \propto \frac{1}{\sqrt{k}}$ (\because m is constant)

31. Phase of 1st oscillator $\theta_1 = \omega_1 t + \phi = \frac{2\pi}{T_1} t + \phi$

For 2nd oscillator, $\theta_2 = \omega_2 t + \phi = \frac{2\pi}{T_2} t + \phi$

phase diff $\theta_1 - \theta_2$

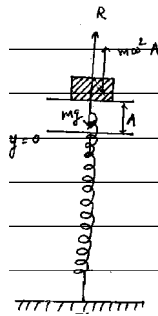
32. Restoring force $F = -A\rho g = -(A\rho g)y = -ky$

$$\therefore k = A\rho g \Rightarrow T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T \propto \frac{1}{\sqrt{A}}$$

33. To loose contact, the condition is ;

$$m\omega^2 A = mg$$

$$\therefore A = \frac{g}{\omega^2} = \frac{mg}{k} (\because k = m\omega^2)$$



34. In SHM, acceleration and displacement are opposite in direction Also $a \propto y$.

35. Here $t = 0$, $x = 1 \text{ cm}$ and $v = \pi \text{ cm s}^{-1}$, $\omega = \pi \text{ s}^{-1}$

$$\text{Now, } x = A \cos(\omega t + \phi) \text{ ---- (1)}$$

$$\text{Velocity } v = \frac{dx}{dt} = -A\omega \sin(\omega t + \phi) \text{ ----- (2)}$$

Solved the equation (1) and (2)

36. $T_1 = 2\pi\sqrt{\frac{144}{g}}$ and $T_2 = 2\pi\sqrt{\frac{121}{g}} \therefore T_1 > T_2$

\therefore When the shorter pendulum completes n oscillations, the longer one completes $(n-1)$ oscillations (when in same phase).

$$\therefore nT_2 = (n-1)T_1$$

37. $\therefore \omega = \frac{\omega^2 r}{r\omega} = 3.14 \therefore 2\pi f = 3.14 \Rightarrow f = \frac{3.14}{2\pi} = 0.5 \text{ s}^{-1}$

38. Maximum force $= m\omega^2 A = m4\pi^2 f^2 A$

39. Periodic time $T = 2\pi\sqrt{\frac{l}{g}}$ and $\omega = \frac{2\pi}{T} \Rightarrow \omega = \sqrt{\frac{g}{l}}$

Linear displacement $x = a \cos \omega t$

40. $\omega = \sqrt{\frac{k}{m_1 + m_2}}$ or removing m, angular frequency $\omega' = \sqrt{\frac{k}{m_2}}$

41. Kinetic energy $K = \frac{1}{2} m \omega^2 (A^2 - y^2)$

$$\text{Now total energy } E = \frac{1}{2} m \omega^2 A^2$$

42. $y = a \sin \omega t + b \cos \omega t$,

Taking $a = A \cos \theta$ and $b = A \sin \theta$,

$$\begin{aligned} y &= A \cos \theta \sin \omega t + A \sin \theta \cdot \cos \omega t \\ &= A \sin(\omega t + \theta) \end{aligned}$$

Now $a^2 + b^2 = A^2 \therefore A = \sqrt{a^2 + b^2}$.

43. The body will not loose contact with the surface,

if $mg = m\omega^2 r = \frac{m4\pi^2}{T^2} \cdot r$ {where r is amplitude} $\therefore T = 2\pi \sqrt{\frac{r}{g}}$

44. Maximum kinetic energy $K_0 = \frac{1}{2} m \omega^2 A^2 \therefore A = \left(\frac{2K_0}{m\omega^2} \right)^{1/2}$

\therefore Equation for displacement is ;

$$y = A \sin \omega t = \left(\frac{2K_0}{m\omega^2} \right)^{1/2} \sin \omega t$$

45. $E = \frac{1}{2} m \omega^2 A^2 \Rightarrow E \propto \omega^2 A^2$

$$\therefore E \propto (A\omega)^2 \Rightarrow (\omega_1 A_1)^2 = (\omega_2 A_2)^2$$

46. k_2 be the spring constant of the spring having length ℓ_2 .

Now, $\ell_1 + \ell_2 = \ell$

$$n\ell_2 + \ell_2 = \ell$$

47. $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ and $f' = \frac{1}{2\pi} \sqrt{\frac{2k}{2m}}$ { $\therefore k' = 2k$ }

$$\therefore f' = f$$

48. Here both springs are in parallel. The restoring force on the system is only due to spring and not due to gravitational force \therefore We can ignore the slope.

Equivalent spring constant $= k + k = 2k$

$$\therefore \text{Periodic time } T = 2\pi\sqrt{\frac{M}{2k}}$$

49. Energy stored = Work done

$$\therefore E = \frac{1}{2}kA^2$$

Now maximum acceleration

$$a_{\max} = \omega^2 A$$

50. Potential energy gained by the spring on suspending mass "m" is $\frac{1}{2}ky^2$.

When system executes SHM, the energy gained by the system $= \frac{1}{2}m\omega^2 A^2$

$$\therefore \text{total final energy of the system} = \frac{1}{2}m\omega^2 A^2 + \frac{1}{2}ky^2.$$

51. Radius of the rotational motion $r = 0.4 \text{ m}$

When the turn table rotates, the restoring force developed in the spring = centrifugal force

$$\therefore F_{\text{restore}} = m\omega^2 r = 2(10)^2 \times 0.4 = 80 \text{ N}$$

Now increase in length of spring $= 40 - 35 = 5 \text{ cm}$

$$\therefore \text{Force constant } k = \frac{F}{x} = \frac{80}{0.05} = 1.6 \times 10^3 \text{ N/m.}$$

52. In case-I, springs are connected in parallel.

$$\therefore \text{equivalent force constant } k_p = k_1 + k_2 = 2k.$$

$$\therefore \text{Periodic time } T_p = 2\pi\sqrt{\frac{m}{k_p}} = 2\pi\sqrt{\frac{m}{2k}}$$

In case-II, springs are connected in series.

$$\therefore \text{Equivalent force constant } k_s = \frac{k_1 k_2}{k_1 + k_2} = \frac{k}{2}$$

$$\therefore \text{periodic time } T_s = 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{2m}{k}} \quad \therefore \frac{T_p}{T_s} = \frac{1}{2}.$$

53. Here $T=6$ s

Amplitude $OB = OC = \frac{1}{2} BC = 10$ cm

$\therefore OD = 5$ cm

Now displacement $x = A \sin(\omega t + \phi) \rightarrow (1)$

where $A=10$ cm, $\omega = \frac{2\pi}{T} = \frac{\pi}{3}$ rad

Now if at $t=0$, oscillator is at C.

i.e. at $t=0$, $x=A$

$\therefore A = A \sin(\omega \times 0 + \phi)$ (OR) $A = A \sin \phi$

$\Rightarrow \sin \phi = 1$

$\Rightarrow \phi = \frac{\pi}{2}$

Putting this in eqn. (1)

$x = A \sin(\omega t + \frac{\pi}{2}) = A \cos \omega t = 10 \cos \omega t$

\therefore for $x = 5$ cm,

$5 = 10 \cos \omega t \Rightarrow \cos \omega t = \frac{1}{2}$

$\therefore \omega t = \frac{\pi}{3}$

$\therefore t = 1$ S

54. Force responsible for oscillation in

$F = mg \sin \theta = mg \theta$ { $\because \theta$ is small }

$= mg \cdot \frac{x}{R}$

Comparing this with

$F = -kx$;

$k = \frac{mg}{R}$

55. Let the rod be pressed down by "x" at point A and released.

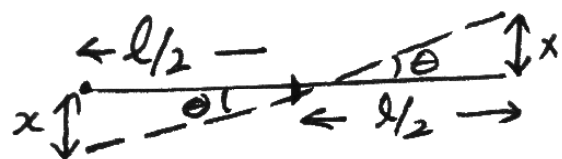
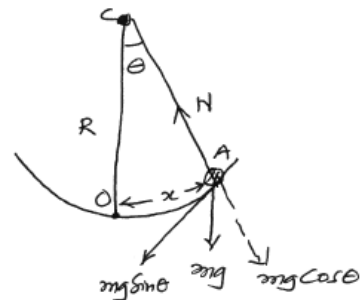
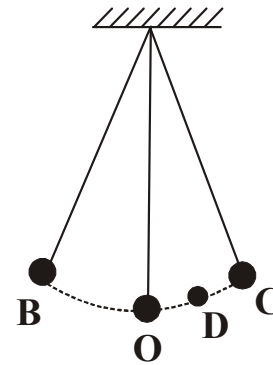
\therefore both spring gets displaced by "x"

\therefore Restoring torque produced

$\tau = kx \times \left(\frac{\ell}{2}\right) + kx \times \left(\frac{\ell}{2}\right)$

$= kx \ell$

Now $\tan \theta = \frac{x/\ell}{\ell/2} = \frac{2x}{\ell}$



If θ is small; $\tan \theta \approx \theta = \frac{2x}{\ell}$

$$\therefore x = \frac{\ell\theta}{2}$$

$$\therefore \text{torque } \tau = k \left(\frac{\ell\theta}{2} \right) \times \ell = \frac{k\ell^2\theta}{2}$$

Now moment of inertia of rod with reference to O is I , then

$$\frac{I d^2\theta}{dt^2} = - \left(\frac{k\ell^2}{2} \right) \theta \quad \therefore \frac{d^2\theta}{dt^2} = - \left(\frac{k\ell^2}{2I} \right) \theta$$

Comparing with $\therefore \frac{d^2\theta}{dt^2} = -\omega^2\theta$;

$$\omega = \sqrt{\frac{k\ell^2}{2I}} \quad \text{where } \omega = \frac{2\pi}{T} \text{ and } I = \frac{m\ell^2}{12}$$

$$\therefore T = \pi \sqrt{\frac{2m}{3k}}$$

56. Here 2 acceleration vectors g and a are acting along mutually perpendicular direction .

$$\therefore \text{effective acceleration in } l^n \quad g_{\text{eff}} = \sqrt{g^2 + a^2}$$

$$\therefore T = 2\pi \sqrt{\frac{\ell}{g_{\text{eff}}}}$$

57. $g_{\text{eff}}^2 = a_x^2 + (g - a_y)^2$ here, $a_x = g \sin \theta \cos \theta$, $a_y = g \sin^2 \theta$

$$= a_x^2 + g^2 + a_y^2 - 2ga_y$$

$$= a^2 \sin^2 \theta \cos^2 \theta + g^2 + g^2 \sin^2 \theta - 2g^2 \sin^2 \theta$$

$$= g^2 (1 - \sin^2 \theta)$$

$$= g^2 \cos^2 \theta$$

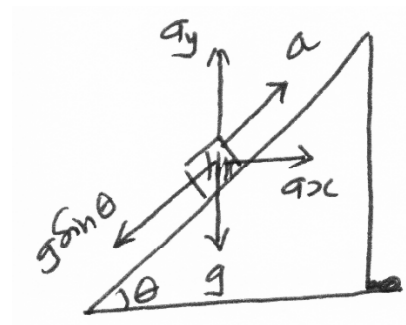
$$\therefore g_{\text{eff}} = g \cos \theta$$

58. When the length of spring increases by $x=2.5$ cm

$$\text{force } F = mg \sin \theta$$

$$\therefore \text{force constant } k = \frac{F}{x} = \frac{mg \sin \theta}{x}$$

$$\therefore T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{x}{g \sin \theta}} = \frac{\pi}{7} \text{ s.}$$



59. Since the spring is massless, when C collides with A, both A and B will gain equal momentum. Also, since A and B have equal mass, both will have same velocity. Let this velocity be u .

\therefore Acc. to the law of conservation of momentum,

$$mv = mu + mu = 2mu$$

$$\therefore u = \frac{v}{2}$$

Now if the compression produced in the spring is x , then acc. to law of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + \frac{1}{2}mu^2 + \frac{1}{2}kx^2$$

$$\therefore v^2 = 2u^2 + \frac{kx^2}{m} = 2\left(\frac{v^2}{4}\right) + \frac{kx^2}{m}$$

$$\therefore \frac{kx^2}{m} = \frac{v^2}{2} \Rightarrow x = v\sqrt{\frac{m}{2k}} \text{ ----- (1)}$$

Now block A and B will have equal kinetic energy.

$$\therefore \frac{1}{2}kx^2 = \frac{1}{2}mu^2 + \frac{1}{2}mu^2 = mu^2$$

\therefore During maximum contraction, kinetic energy of the system A-B is

$$mu^2 = \frac{m \cdot v^2}{4}$$

60. Displacement

$$y = 4 \cos^2\left(\frac{t}{2}\right) \sin 100t$$

61. Here $x = A \cos \omega t$

Now potential energy = $\frac{1}{2}m\omega^2x^2$ {taking P.E. as a function of x }

\therefore when $x=0$, potential energy=0

\therefore graph (b) \rightarrow III

Also, potential energy = $\frac{1}{2}m\omega^2(A \cos \omega t)^2$ {taking P. E. as a function of time}

At $t=0$ potential energy = $\frac{1}{2}m\omega^2A^2$

\therefore graph -I

$$62. E_1 = \frac{1}{2} m \omega^2 x^2 \Rightarrow \sqrt{E_1} = x \sqrt{\frac{1}{2} m \omega^2} \text{-----(1)}$$

$$E_2 = \frac{1}{2} m \omega^2 y^2 \Rightarrow \sqrt{E_2} = y \sqrt{\frac{1}{2} m \omega^2} \text{-----(2)}$$

$$\therefore E = \frac{1}{2} m \omega^2 (x + y)^2 \Rightarrow \sqrt{E} = (x + y) \sqrt{\frac{1}{2} m \omega^2} \text{-----(3)}$$

From (1), (2) & (3),

$$\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$$

$$\text{or } E = E_1 + E_2 + 2\sqrt{E_1 E_2}$$

$$63. \omega_1^2 = \frac{k}{m} = \frac{kx}{mx} = \frac{F_1}{mx} \text{-----(1)}$$

$$\text{Similarly, } \omega_2^2 = \frac{F_2}{mx} \text{-----(2)}$$

If F_1 and F_2 acts simultaneously, then angular frequency

$$\omega_2 = \frac{F_1 + F_2}{mx} \text{-----(3)}$$

From (1), (2) and (3); $\omega^2 = \omega_1^2 + \omega_2^2$ Now, use equ. $\omega = \frac{2\pi}{T}$

$$64. \text{Initial periodic time } T_1 = 2\pi \sqrt{\frac{\ell}{g}} \rightarrow (1)$$

When pendulum moves along vertical direction, effective acceleration $g_{\text{eff}} = g + a$

where 'a' in acceleration of pendulum.

$$\text{Now, } a = \frac{dv}{dt} = \frac{d(kt)}{dt} = k = 2.1 \text{ m.s}^{-2}$$

$$\therefore \text{New periodic time } T_2 = 2\pi \sqrt{\frac{\ell}{g_{\text{eff}}}} \rightarrow (2)$$

$$\therefore \frac{T_2}{T_1} = \sqrt{\frac{g}{g_{\text{eff}}}}$$

65. Block will not slide if $\mu mg \geq ma$

$$\Rightarrow \mu g \geq a$$

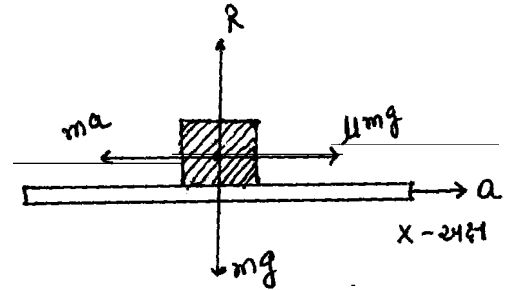
To prevent the block from sliding the maximum acceleration

of table must be $a_{\max} = \mu g$

Now maximum acceleration $a_{\max} = \omega^2 A$

$$\therefore \omega^2 A_{\max} = \mu g$$

$$\therefore A_{\max} = \frac{\mu g}{\omega^2} = \frac{\mu g T^2}{4\pi^2}$$



66. Angular frequency of system

$$\omega = \left[\frac{k}{(M + m)} \right]^{1/2} \text{-----(1)}$$

Now to prevent B from sliding off A, the maximum force acting on B should not be more than the frictional force μmg .

$$\therefore f_{\max} = ma_{\max} = m\omega^2 A_{\max} \longrightarrow (2)$$

From (1) & (2)

$$\therefore f_{\max} = m \left(\frac{k}{m + M} \right) A_{\max}$$

To prevent block from sliding, $\therefore f_{\max} = \mu mg \therefore \frac{mkA_{\max}}{m + M} = \mu mg$

67. Restoring force $F = -kx$

$$\text{Now, } F = -\frac{du}{dx} \quad \therefore -kx = -\frac{du}{dx} \quad \therefore du = k \cdot x \cdot dx$$

$$\therefore U(x) = \int_0^x k \cdot x \cdot dx = \frac{kx^2}{2} + C$$

Where C is constant of integration.

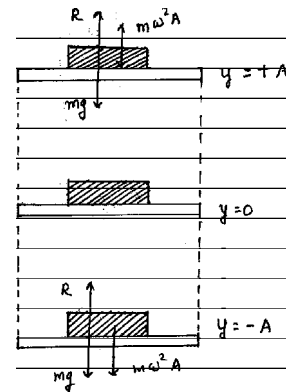
Now in a SHM, potential energy at the equilibrium position is zero.

$$\therefore u(x = 0) = 0 \quad \therefore C = 0$$

$$u(x) = \frac{1}{2} kx^2 \text{ in an equation for a parabola.}$$

68. At the upper most end,
 when $mg = R + m\omega^2 A$, coin will loose contact.
 Taking $R=0$
 $m\omega^2 A = mg$

$$A = \frac{g}{\omega^2}$$



69. Force required to increase the length by x in $F=kx$ -----(1)

After spring is divided into 2 equal parts,

$$F = k'x' \quad \text{where } x' = \frac{x}{2}$$

$$= k' \frac{x}{2} \text{-----(2)}$$

from (1) & (2); $k' = 2k$

70. Frequency of SHM depends on elasticity & inertia.

71. Restoring force $F = -mg \sin \theta$ OR

$$F = -mg_e \quad \text{where } g_e = g \sin \theta$$

If θ is small, $\sin \theta \cong \theta$

\therefore Effective value of g is $g_e \theta$

For large oscillation, $g \sin \theta < g \theta$ ($\because \sin \theta < \theta$)

$$\therefore T > 2\pi \sqrt{\frac{\ell^1}{g}}$$

72. Restoring force $F = -mg \sin \theta$

which depends on "m"

73. "g" is less on moon

$$\therefore \text{form the equation } T = 2\pi \sqrt{\frac{\ell^1}{g}},$$

T will increase

As compared to earth, moon is small

74. Periodic time $T \propto \sqrt{\ell}$

$$\therefore \Delta T = \frac{1}{2\sqrt{\ell}} \cdot \Delta \ell \quad \{\text{On differentiation}\}$$

$$\therefore \frac{\Delta T}{T} = 1.5\%$$

75. $a_{\max} = \omega^2 A = 4\pi^2 f^2 A = 4\pi^2 (30)^2 \times 0.01$

$$= 36\pi^2 \text{ m/s}^2$$

Since the oscillator moves between $+A$ & $-A$,

$$\text{maximum acceleration} = \pm 36\pi^2$$

77. Energy dissipates & so amplitude decreases.

Statement -2 is false.

78. Statement -1 is true. statement -2 is false. In a SHM, amplitude & phase does not depend on restoring force.

79. Time taken by the spring k_2 to get maximum compressed from point D = half period of oscillation of the block.

(if block is attached at the free end of spring)

$$\text{i.e. } t_2 = \frac{T_2}{2} = \frac{1}{2} 2\pi \sqrt{\frac{m}{k_2}} = \frac{1}{2} 2\pi \sqrt{\frac{0.2}{0.3}} = \frac{\pi}{4} \text{ s}$$

80. Similarly $t_1 = \frac{T_1}{2} = \frac{\pi}{3} \text{ s}$

81. Time period of Block T = Time taken by the block to move from C to D and D to C

$$82. \quad f = \frac{1}{2\pi} \sqrt{\frac{k}{M}} \quad \text{and} \quad f' = \frac{1}{2\pi} \sqrt{\frac{k}{m+M}}$$

83. According to the law of conservation of momentum,

$$MV = (M+m)v'$$

84. According to the law of conservation of energy,

Kinetic Energy at mid point = potential Energy at the end points

$$\therefore \frac{1}{2} Mv^2 = \frac{1}{2} kA^2$$

$$\text{And } \frac{1}{2} (M+m)v'^2 = \frac{1}{2} kA'^2$$

85. $y = 3 \cos 2t + 4 \sin 2t$

$$\therefore A \sin \phi = 3 \quad \text{And} \quad A \cos \phi = 4$$

$$\therefore y = A \sin \phi \cos 2t + A \cos \phi \sin 2t$$

$$\therefore y = A \sin (\omega t + \phi)$$

Which shows that the motion is simple harmonic motion

86. $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi \text{ s}$

87. Amplitude $A = \sqrt{3^2 + 4^2} = 5 \text{ cm}$

88. Maximum Acceleration of

$$a \text{ particle} = A\omega^2 = 5(2)^2 = 20 \text{ cm s}^{-2}$$

89. Mechanical Energy $= \frac{1}{2} m\omega^2 A^2 = 250 \text{ erg}$

90. frequency of the particle $f = \frac{1}{T} = \frac{1}{\pi} \text{ S}^{-1}$

91. On comparing $y = A \sin (15\pi t + 10\pi x + \frac{\pi}{3})$

$$\text{with } y = A \sin (\omega t + kx + \theta)$$

92. At constant pressure density of water vapour is less than dry air.

$$\therefore \text{with increase in humidity according to the equation } v = \sqrt{\frac{\gamma P}{\rho}} \text{ the}$$

velocity of sound increases.

93. $f \propto \lambda^{-1} \quad \therefore \frac{f_1}{f_2} = \frac{\lambda_2}{\lambda_1}$

94. From the equation $v = f\lambda$, $\lambda_{\min} = \frac{v}{f_{\max}} = 17 \text{ mm}$ which is nearer to 20 mm

95. On comparing with the wave equation

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) \text{ we get, } T = 0.04 \text{ s, } \lambda = 0.5 \text{ m} \Rightarrow v = \frac{25}{2} \text{ ms}^{-1}$$

$$\therefore T = v^2 \pi = 6.25 \text{ N}$$

96. Maximum velocity of particle = $A\omega$

$$\therefore \text{wave velocity} = f\lambda$$

$$\therefore \text{Maximum velocity of particle} = 2 \times \text{wave velocity}$$

$$\therefore A\omega = 2f\lambda \Rightarrow \lambda = \pi A$$

97. Putting values in $\lambda = vT$

if phase diff. = in the interval Δx is $\Delta\delta$ then

$$\Delta\delta = \frac{2\pi}{\lambda} \Delta x = \frac{2\pi}{15} \times (15 - 10) = \frac{2\pi}{3}$$

98. Freq. of a wave in a string $f \propto \frac{1}{\ell}$

$$\therefore \ell = \ell_1 + \ell_2 + \ell_3 \quad \therefore \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

99. On comparing $y_1 = 4 \sin 500\pi t$ with $y_1 = A \sin \omega_1 t$

$$\text{we get } \omega_1 = 2\pi f_1 = 500\pi \Rightarrow f_1 = 250 \text{ Hz}$$

$$\text{Similarly } y_2 = 2 \sin 506\pi t$$

$$\therefore \omega_2 = 2\pi f_2 = 506\pi \Rightarrow f_2 = 253 \text{ Hz}$$

$$\therefore \text{Freq. of beats} = f_2 - f_1 = 3$$

$$\therefore \text{No. of beats heard per minute} = 3 \times 60 = 180$$

100. $y = 8 \sin 2\pi (0.1x - 2t)$

$$\therefore y = -8 \sin 2\pi (2t - 0.1x) \text{ comparing with } y = A \sin \left(\frac{t}{T} - \frac{x}{\lambda} \right)$$

$$\text{We get } \frac{1}{\lambda} = 0.1 \Rightarrow \lambda = 10 \text{ cm}$$

$$\text{Now path difference between 2 particles } \delta = \frac{2\pi}{\lambda} \cdot x = kx$$

$$\therefore \delta = \frac{2 \times 180 \times 2}{10} = 72^\circ$$

101. Distance covered by the pulse = speed \times time = 4 cm in 2 seconds both will cover 4 cm & the centre of both will superpose & potential energy will be zero.

$$\therefore \text{Total energy will be in the form of kinetic energy.}$$

102. $y_1 = a \sin \omega t$ & $y_2 = a \cos \omega t = a \sin(\omega t + \pi/2)$

\therefore 1st wave is lagging behind in phase by $\pi/2$

103. Here A is the amplitude of resultant wave formed by 2 waves having amplitude A_1 and A_2 respectively.

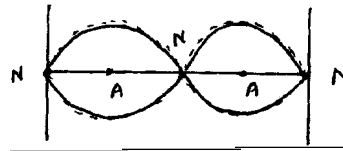
$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos \theta \text{ Also } \theta \text{ is the phase } A_1, A_2$$

Now putting $A_1 = A_2 = b$ & $A = b$, We get

$$b^2 = 2b^2(1 + \cos \theta)$$

$$\therefore \cos \theta = -1/2 \Rightarrow \theta = 120^\circ$$

104. As seen from fig., distance between 3 nodes in λ



105. $y = \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} = \frac{1}{2} - \frac{1}{2} \cos 2\omega t$ _____ (1)

$$\therefore v = \frac{1}{2} 2\omega \sin(2\omega t) = \omega \sin 2\omega t$$

$$\therefore a = 2\omega^2 \cos 2\omega t$$

$$= 2 \times 2\omega^2 \left(\frac{1}{2} - Y \right) \text{ \{From eq. (1)\}}$$

$$= -4\omega^2 \left(\frac{1}{2} - y \right)$$

$$\therefore a \propto -y \text{ \{ \therefore SHM \}}$$

$$\text{Now, } \frac{2\pi}{T} = 2\omega \Rightarrow T = \frac{\pi}{\omega}$$

106. No. of beats produced per second = $n_1 - n_2$

$$\therefore \text{Time interval between 2 consecutive beats} = \frac{1}{n_1 - n_2}$$

107. Since the phase difference between the 2 waves is $\frac{\pi}{2}$ they are oscillating along mutually perpendicular directions.

$$\therefore \text{Resultant amplitude} = \sqrt{A^2 + A^2} = \sqrt{2} A$$

\therefore Angular freq. will remain same.

108. $A = 0.02 \text{ m}$, $v = \frac{\omega}{k} = 128 \text{ ms}^{-1}$

$$5\lambda = 4 \Rightarrow \lambda = \frac{4}{5} \text{ m}, k = \frac{2\pi}{\lambda} = 2.5\pi = 7.85$$

$$\therefore \omega = 128 \times k = 128 \times 7.85 \cong 1005$$

$$y = A \sin(kx - \omega t)$$

$$\therefore y = 0.02 \sin(7.85x - 1005t)$$

109. Let the number of loops obtained for 315Hz and 420Hz n and $(n+1)$ respectively.

$$\therefore f_n = nf_1 = 315$$

$$\therefore f_{n+1} = (n+1)f_1 = 420$$

$$\therefore f_{n+1} - f_n = f_1 = 105 \text{ Hz}$$

110. When, sound waves travel from one medium to another, its frequency does not change.

$$\therefore f = \frac{v}{\lambda} = \text{constant}$$

$$\therefore \frac{v_a}{\lambda_a} = \frac{v_b}{\lambda_b}$$

$$\lambda_b = \frac{v_b}{v_a} \lambda_a = 10\lambda_a$$

111. Wave velocity = max. velo. of particle

$$\frac{\omega}{k} = A\omega \quad \therefore A = \frac{1}{k} = \frac{\lambda}{2\pi}$$

$$\therefore \lambda = 2\pi A$$

112. Speed of sound in an ideal gas $v = \sqrt{\frac{\gamma RT}{m}}$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}} \quad \left(\because v \propto \frac{1}{\sqrt{m}} \right)$$

113. $E = \frac{1}{2} m \omega^2 A^2 = \frac{1}{2} m \quad 4\pi^2 f^2 A^2$

$$\therefore E \propto f^2 \Rightarrow \frac{E_1}{E_2} = \left(\frac{f_1}{f_2} \right)^2 = \left(\frac{f}{2f} \right)^2 = \frac{1}{4} \quad \therefore E_2 = 4E_1$$

114. Let the freq. of 1st fork be f_1

$$\therefore \text{frequency of 2nd fork} = f_1 + 6 = f_1 + 6(2-1)$$

$$\therefore \text{freq. of 24th fork} = f_1 + 6(24-1) = f_1 + 138$$

Now, freq. of 24th fork = 2 x freq. of 1st fork (given)

$$\therefore f_1 + 138 = 2 f_1 \quad \therefore f_1 = 138 \text{ Hz}$$

115. Differentiating $y_1 = 0.1 \sin\left(100t + \frac{\pi}{3}\right)$ w.r.t time,

$$v_1 = (0.1)(100) \cos\left(100\pi t + \frac{\pi}{3}\right)$$

Similarly differentiating $y_2 = 0.1 \cos \omega t$ w.r.t. time,

$$v_2 = 0.1 \sin\left(\pi t + \frac{\pi}{2}\right)$$

\therefore phase difference between the 2 velocities is

$$\delta = \left(\pi t + \frac{\pi}{3}\right) - \left(\pi t + \frac{\pi}{2}\right) = \frac{-\pi}{6} \text{ rad}$$

116. Wave number = $\frac{1}{\lambda} = \frac{1}{0.005} = 200 \text{ m}^{-1}$

117. Frequency heard by the listener

$$f_L = \left(\frac{v+v_L}{v}\right) f_s \quad (\because v_s = 0)$$

$$\therefore \frac{f_L}{f_s} = \frac{v+v_2}{v} = \frac{v+\frac{v}{4}}{v} = \frac{5}{4}$$

$$\therefore \% \text{ increase} = \frac{f_L - f_s}{f_s} \times 100 = \left(\frac{5-4}{4}\right) \times 100 = 25\%$$

118. From $v = \sqrt{\frac{\gamma RT}{M}}$, $v \propto \sqrt{T}$

In summer, velocity increases & hence decreases and so L increases.

$$\text{The length of 2nd harmonics } x = 3L_1 = 3 \times 16 = 48 \text{ cm}$$

In summer, velocity being more, $x > 3L_1 \quad \therefore x > 48$

$$119. \quad \text{In } f_L = \left(\frac{v+v_L}{v+v_s} \right) f_s$$

$$\text{putting } v_L = 0, \quad f_L = 2f_s, \quad v = v, \quad v_s = -v_s$$

$$2f_s = \left(\frac{v}{v-v_s} \right) f_s \quad \Rightarrow \quad 2v_s = v \quad \therefore \quad v_s = \frac{v}{2}$$

120. For resonance, the frequency of a.c. supply should be same as fundamnetal freq. of wire.

$$\therefore f = \frac{1}{2L} \sqrt{T} = 50 \text{ Hz}$$

$$121. \quad \frac{f_L}{f_s} = \frac{v+v_L}{v+v_s} \quad \text{or} \quad \frac{f_L}{f_s} = \frac{v-v_L}{v-v_s}$$

$$\text{but, } v_L = v_s \quad \therefore \quad \frac{f_L}{f_s} = 1$$

122. Since the rope is heavy, the tension at the lower end & top end of the rope will be different.

$$\text{Mass of rope } m_2 = 3 \text{ kg}$$

$$\text{Mass of block } m_1 = 1 \text{ kg}$$

$$\therefore \text{ tension at the lower end } T_1 = m_1 g = 1 \text{ g N \&}$$

$$\text{at the upper end in } T_2 = (m_1 + m_2)g = 4 \text{ g N}$$

$$\text{Now speed of wave in rope } v = \sqrt{T} \Rightarrow f \lambda = \sqrt{T}$$

$$\therefore \lambda = \sqrt{T} \quad (\because f, \mu \text{ are constants})$$

$$\therefore \text{ Wave length at lower and } \lambda_1 = \sqrt{T_1} \text{ \& at}$$

$$\text{the upper end } \lambda_2 = \sqrt{T_2}$$

$$\therefore \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow \lambda_2 = \sqrt{\frac{T_2}{T_1}} = \lambda_1 = \lambda_2 = 0.1 \text{ m}$$

$$123. \quad \text{Speed of sound} = \sqrt{\frac{\gamma P}{\rho}}$$

$$\rho = \frac{\text{mass of 1 mole air}}{\text{Volume of 1 mole air}} = \frac{29 \times 10^{-3} \text{ kg}}{22.4 \times 10^{-3} \text{ m}^3} = 1.3$$

$$\therefore \text{ speed} = \sqrt{\frac{7}{5} \times \frac{1.01 \times 10^5}{1.3}} = 330 \text{ ms}^{-1}$$

124 From the phase angle $(40-2t)$, we get $k = 40$ OR $\frac{2\pi}{\lambda} = 40 \Rightarrow \lambda = \frac{\pi}{20}$

and $\omega = 2$ OR $2\pi f = 2 \Rightarrow f = \pi^{-1} \text{ Hz}$

125 Increase in tension of string increases its frequency. If the original frequency of B (f_B) were greater than that of A (f_A), further the increase in f_B should have resulted in increase in the beat frequency. But the beat frequency is found to decrease. This shows that $f_A - f_B = 5 \text{ Hz}$ and $f_A = 427 \text{ Hz}$, we get

$$f_B = 422 \text{ Hz}$$

126
$$f_L = \left(\frac{v - v_L}{v - v_S} \right) f_S = \left[\frac{330 - 0}{330 - 130} \right] \times 800 = 1320 \text{ Hz}$$

127
$$v = \sqrt{\frac{T}{\mu}} \Rightarrow T = \mu v^2 = \frac{M}{L} v^2 = \frac{2.2}{11} \times (340)^2 \therefore T = 2.31 \times 10^4 \text{ N}$$

128
$$f = \frac{1}{2l} \sqrt{\frac{T}{M}} \Rightarrow T = f^2 4l^2$$

$$\therefore T = 4f^2 \left(\frac{M}{4} \right)^2 = \frac{4f^2 M^2}{4} = 248 \text{ N}$$

129 In tube A, $\lambda_A = 2l$

In tube B, $\lambda_B = 4l$

$$\therefore v_A = \frac{v}{\lambda_A} = \frac{v}{2l} \quad \therefore v_B = \frac{v}{\lambda_B} = \frac{v}{4l} \Rightarrow \frac{v_A}{v_B} = \frac{2}{1}$$

130 The was decreases the frequency of unknown fork. The possible unknown frequencies are, $(288+4) \text{ Hz}$ and $(288-4) \text{ Hz}$. Wax reduces 284 Hz and so beats should increases. It is not given in the question. This frequency is ruled out. Wax reduces 292 Hz and so beats should decrease. It is given that the beats decrease from 2 to 4. Hence the unknown fork has frequency 292 Hz .

consider option (a)

131 Stationary wave : $Y = a \sin (wt-kx) + a \sin (wt+kx)$

When $x = 0$, $Y \neq 0$. The option is not acceptable

consider option (b)

stationary wave : $Y = a \sin (wt-kx) - a \sin (wt+kx)$

At $x = 0$, $Y = 0$. This option holds good.

Option (c) gives $Y = 2a \sin (wt - kx)$

At $x = 0$, $Y \neq 0$

Option (d) gives $Y = 0$.

Hence option (b) holds good.

132 When temperature increases, l increases. Hence frequency decreases.

133 The possible frequency of piano are $(256 + 5)\text{Hz}$ and $(256 - 5)\text{Hz}$.

For a piano string $v = \frac{1}{2l} \sqrt{\frac{T}{\rho}}$ When tension T increases v increases.

(i) If 261 Hz increases, beats / second increase. This is not given.

(ii) If 251 Hz increases due to tension, beats / second decrease. This is given.

134 By Doppler's effect, $\frac{f_L}{f_s} = \left(\frac{v + v_L}{v + v_s} \right) \therefore \frac{f_L}{f_s} = \frac{v + v_L}{v} = \frac{v + v/5}{v} = \frac{6}{5}$

$$\therefore \text{Fractional increase} = \frac{f_L - f_s}{f_s} = \frac{f_L}{f_s} - 1 = \frac{6}{5} - 1 = \frac{1}{5}$$

$$\therefore \text{percentage increase} = \frac{100}{5} = 20\%$$

135 $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

$$\therefore \frac{v_2}{v_1} = \sqrt{\frac{\gamma_{\text{He}}}{4} \times \frac{32}{\gamma_{\text{O}_2}}}$$

Hence option (d) is correct.

136 In a longitudinal wave, pressure is maximum where displacement is minimum. Therefore pressure and displacement variations are 180° out of phase

137 Frequency of tuning fork $f_1 = 480\text{ Hz}$. Number of beats s^{-1} , $n = 10$

Frequency of string $f_2 = (480 + 10)\text{Hz}$. A slight increase in tension increase f_2
 $f_2 = 480 - 10 = 470\text{ Hz}$.

138 (c) is the correct choice because its value is finite at all times.

139 As $\sin(90 \pm \theta) = \cos \theta \therefore$ The phase difference between the two waves is $\frac{\pi}{2}$

140 $v = \sqrt{\frac{T}{\rho}} = \sqrt{\frac{500}{0.2}} = 50\text{ms}^{-1}$

141 Here, $\rho_1 = \rho_2$, $\frac{r_1}{r_2} = \frac{1}{2}$, $T_1 = T_2$

$$f_1 = \frac{1}{2l r_1} \sqrt{\frac{T_1}{\rho_1}}, f_2 = \frac{1}{2l r_2} \sqrt{\frac{T_2}{\rho_2}} \therefore \frac{f_1}{f_2} = \frac{r_1}{r_2} = \frac{2}{1}$$

$$142 \quad \frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{81}{100}} = \frac{9}{10} \therefore \frac{f_1 - f_2}{f_1} \times 100 = 10\%$$

$$143 \quad \text{When one end is closed } f_1 = \frac{100}{2} = 50\text{Hz}$$

$f_2 = 3f_1 = 150\text{Hz}, f_3 = 5f_1 = 250\text{Hz}$ and so on...

144 When other end of pipe is opened, its fundamental frequency becomes 200Hz. The overtone have frequencies 400, 600, 800 Hz..

$$145 \quad \text{As } \frac{\ell_2}{2\ell} = \frac{\ell_2'}{\ell_1'} \Rightarrow \frac{60}{40} = \frac{\ell_2'}{50} \Rightarrow \ell_2' = 75\text{cm}$$

$$146 \quad \frac{f_1}{f_2} = \frac{\ell_2}{\ell_1} = \frac{25}{30} = \frac{5}{6}$$

$f_2 - f_1 = 4$. on solving, we get $f_2 = 24$ Hz

$\therefore f_1 = 20\text{Hz}$

$$147 \quad \frac{f_2}{f_1} = \sqrt{\frac{101}{100}} = \left(1 + \frac{1}{100}\right)^{1/2} = 1 + \frac{1}{200}$$

$$\therefore f_2 = f_1 + \frac{f_1}{200} \therefore \text{numbers of beats}^{-1} = f_2 - f_1 = \frac{f_1}{200} = 1$$

$$148 \quad \frac{f_L}{f_S} = \frac{v + v_L}{v + v_S}, \text{ Here } v_L = +5 \text{ ms}^{-1}, v_S = -5 \text{ ms}^{-1}, f_S = 165 \text{ Hz}$$

$$\therefore f_L = 170 \text{ Hz} \therefore \text{Number of beats}^{-1} = 170 - 165 = 5$$

149 As the source is moving perpendicular to straight line joining the observer and source, (as if moving along a circle), apparent frequency is not affected $n_1 = 0$

150 As is clear from figure, at $t = 0, x = 0$, displacement $y = 0$ Therefore option (A) OR (D) may be correct. In case of (D) $y = A \sin(kx - \omega t)$

$$\therefore \frac{dy}{dt} = -\omega A \cos(kx - \omega t) \quad \frac{dy}{dx} = kA \cos(kx - \omega t) \quad \therefore \frac{\frac{dy}{dt}}{\frac{dy}{dx}} = -v \Rightarrow \frac{dy}{dt} = -v \frac{dy}{dx}$$

i.e. particle velocity = - (wave speed) x slope and slope at $x = 0$ and $t = 0$ is positive, in figure Therefore, particle velocity is in negative y - direction

151 At a displacement antinode, a pressure node is present. Since pressure does not change at its node, nor does density.

152 For a sonometer fundamental $f = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$

To maintain the fundamental mode, in doubling the length, tension must be quadrupled.

153 velocity of transverse waves $v_T = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{\pi r^2 \rho}}$

velocity of longitudinal waves $v_L = \sqrt{\frac{Y}{\rho}}$

$$\therefore \frac{v_L}{v_T} = \sqrt{\frac{Y}{T/\pi r^2}} = \sqrt{\frac{Y}{\text{stress}}}$$

154 Let the frequency of standard fork = x

$$\therefore f_A = \frac{102}{100}x, f_B = \frac{97}{100}x, f_C = \frac{97}{100}x$$

$$\text{Now } f_A - f_B = \frac{102x}{100} - \frac{97x}{100} \therefore x = 120\text{Hz}$$

155 If the length of the wire between the two bridges is l , then the frequency of vibration is

$$n = \frac{1}{2l} \sqrt{\frac{T}{m}} = \frac{1}{2l} \sqrt{\frac{T}{\pi r^2 d}}$$

If the length and diameter of the wire are doubled keeping the tension same, then new fundamental frequency will be $n/4$

156 $\frac{f_L}{f_S} = \frac{v + v_L}{v + v_S}$ using this equation the frequency of reflected sound heard by the girl,

$$f_L = \frac{v + v_L}{v - v_S} f_S$$

157 $f_{\text{open}} = \frac{v}{2l_{\text{open}}}$

$$f_{\text{closed}} = \frac{v}{4l_{\text{closed}}} = \frac{v}{4l_{\text{open}}/2} \left[\text{As } l_{\text{closed}} = \frac{l_{\text{open}}}{2} \right]$$

$$= \frac{v}{2l_{\text{open}}} = f_{\text{open}}$$

i.e. frequency remains unchanged.

158 If we assume that all the three waves are in same phase at $t = 0$, we shall hear only 1 beat s^{-1}

159 $y(x, t) = 0.005 \cos(\alpha x - \beta t)$ compare it with standard equation

$$y(x, t) = A \cos(kx - \omega t) = A \cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right) \therefore \alpha = \frac{2\pi}{\lambda} \text{ and } \beta = \frac{2\pi}{T}$$

160 Given that the displacement of particle is $y = A \sin(\omega t - kx)$ (i)

The particle velocity $v_p = \frac{dy}{dt}$ (ii) \neq

Now, on differentiating eqn.1 with respect to t $\frac{dy}{dt} = A \omega \cos(\omega t - kx)$

From eqn.(2) fundamental mode of the closed pipe is

$$f_1 = \frac{v}{4L} = \frac{320}{4 \times 0.40} = 200 \text{ Hz}$$

173 Since the beat frequency is 8, the frequency of the string vibrating in its first Overtone is 192 Hz or 208 Hz.

Where for 1st Overtone frequency $f_1' = \frac{1}{\ell} \sqrt{\frac{T}{m}}$ (1)

It is given that the beat frequency decreases if the tension in the string is decreased.

$\therefore f_1' > f_1$ Hence $f_1' = 208 \text{ Hz}$ and not 192 Hz

174 substituting the values of ℓ, m and f_1' in equation 1 we get $T = 27.04 \text{ N}$

175 $\frac{2\pi}{\lambda} = 2 \Rightarrow \lambda = \pi \text{ m}$

$$\frac{2\pi f}{\lambda} = 3\pi \Rightarrow v = \frac{3\lambda}{2} \text{ ms}^{-1}$$

176 Distance between two consecutive nodes $= \frac{\lambda}{2} = \frac{\pi}{2} \text{ m}$

177 The resultant displacement is given by,
 $y = 0.1 \cos 2x \sin 3\pi t$ Or $y = A \sin 3\pi t$

Where A is the Amplitude of standing waves given by $0.1 \cos 2x$

At $x = 0.5 \text{ m}$, $\cos 2x = \cos(1 \text{ rad}) = \cos\left(\frac{\pi}{3.14}\right) = \cos 57.3^\circ = 0.54 \text{ m}$

Amplitude A at $(x = 0.5 \text{ m}) = 0.1 \times 0.54 = 0.54 \text{ m}$

178 Particle velocity $v = \frac{dy}{dt} = \frac{d}{dt}(0.1 \cos 2x \sin 3\pi t) = 0.1 \times 3\pi \cos 2x \sin 3\pi t$

at $x = 0.25 \text{ m}$ and $t = 0.5 \text{ s}$, $v = 0$

179 The two displacements can be written as

$$y_1 = A \cos(k_1 x - \omega_1 t)$$

and $y_2 = A \cos(k_2 x - \omega_2 t)$

compare this equation with given equation and get solution.

180 Beat frequency = $f_1 - f_2 = \frac{\omega_1}{2\pi} - \frac{\omega_2}{2\pi}$

181 The resultant displacement is given by

$$y = y_1 + y_2$$

$$= A \cos(k_1 x - \omega_1 t) + A \cos(k_2 x - \omega_2 t)$$
 For $x = 0$ we have

$$y = A \cos \omega_1 t + A \cos \omega_2 t$$

$$\therefore y = 0.10 \cos(96\pi t) \cos(4\pi t)$$

Between $t = 0$ and $t = 1$ s, $\cos 96\pi t$ becomes zero 96 times and $\cos 4\pi t$ becomes zero 4 times. Hence the resultant displacement Y at $x = 0$ becomes zero 100 times between $t = 0$ and $t = 1$ s.

182 $y_l = A \sin(kx + \omega t)$

$$y_r = A \sin(kx - \omega t) \Rightarrow y = y_l + y_r$$

$$\therefore y = 2A \sin kx \cos \omega t$$
 Here $2A = 10 \therefore A = 5$