

## Quadratic Equations

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### Question 1.

Find which of the following equations are quadratic:

#### Solution 1(i)

$$(3x - 1)^2 = 5(x + 8)$$

$$\Rightarrow (9x^2 - 6x + 1) = 5x + 40$$

$$\Rightarrow 9x^2 - 11x - 39 = 0; \text{ which is of the form } ax^2 + bx + c = 0.$$

$\therefore$  Given equation is a quadratic equation.

#### Solution 1(ii)

$$5x^2 - 8x = -3(7 - 2x)$$

$$\Rightarrow 5x^2 - 8x = 6x - 21$$

$$\Rightarrow 5x^2 - 14x + 21 = 0; \text{ which is of the form } ax^2 + bx + c = 0.$$

$\therefore$  Given equation is a quadratic equation.

#### Solution 1(iii)

$$(x - 4)(3x + 1) = (3x - 1)(x + 2)$$

$$\Rightarrow 3x^2 + x - 12x - 4 = 3x^2 + 6x - x - 2$$

$$\Rightarrow 16x + 2 = 0; \text{ which is not of the form } ax^2 + bx + c = 0.$$

$\therefore$  Given equation is not a quadratic equation.

#### Solution 1(iv)

$$x^2 + 5x - 5 = (x - 3)^2$$

$$\Rightarrow x^2 + 5x - 5 = x^2 - 6x + 9$$

$$\Rightarrow 11x - 14 = 0; \text{ which is not of the form } ax^2 + bx + c = 0.$$

$\therefore$  Given equation is not a quadratic equation.

#### Solution 1(v)

$$7x^3 - 2x^2 + 10 = (2x - 5)^2$$

$$\Rightarrow 7x^3 - 2x^2 + 10 = 4x^2 - 20x + 25$$

$$\Rightarrow 7x^3 - 6x^2 + 20x - 15 = 0; \text{ which is not of the form } ax^2 + bx + c = 0.$$

$\therefore$  Given equation is not a quadratic equation.

#### Solution 1(vi)

$$(x - 1)^2 + (x + 2)^2 + 3(x + 1) = 0$$

$$\Rightarrow x^2 - 2x + 1 + x^2 + 4x + 4 + 3x + 3 = 0$$

$$\Rightarrow 2x^2 + 5x + 8 = 0; \text{ which is of the form } ax^2 + bx + c = 0.$$

$\therefore$  Given equation is a quadratic equation.

**Question 2(i)**

Is  $x = 5$  a solution of the quadratic equation  $x^2 - 2x - 15 = 0$ ?

**Solution:**

$$x^2 - 2x - 15 = 0$$

For  $x = 5$  to be solution of the given quadratic equation it should satisfy the equation.

So, substituting  $x = 5$  in the given equation, we get

$$\text{L.H.S} = (5)^2 - 2(5) - 15$$

$$= 25 - 10 - 15$$

$$= 0$$

$$= \text{R.H.S}$$

Hence,  $x = 5$  is a solution of the quadratic equation  $x^2 - 2x - 15 = 0$ .

**Question 2(ii).**

Is  $x = -3$  a solution of the quadratic equation  $2x^2 - 7x + 9 = 0$ ?

**Solution:**

$$2x^2 - 7x + 9 = 0$$

For  $x = -3$  to be solution of the given quadratic equation it should satisfy the equation

So, substituting  $x = -3$  in the given equation, we get

$$\text{L.H.S} = 2(-3)^2 - 7(-3) + 9$$

$$= 18 + 21 + 9$$

$$= 48$$

$$\neq \text{R.H.S}$$

Hence,  $x = -3$  is not a solution of the quadratic equation  $2x^2 - 7x + 9 = 0$ .

**Question 3.**

If  $\sqrt{\frac{2}{3}}$  is a solution of equation  $3x^2 + mx + 2 = 0$ , find the value of  $m$ .

**Solution:**

For  $x = \sqrt{\frac{2}{3}}$  to be solution of the given quadratic equation it should satisfy the equation

So, substituting  $x = \sqrt{\frac{2}{3}}$  in the given equation, we get

$$3\left(\sqrt{\frac{2}{3}}\right)^2 + m\left(\sqrt{\frac{2}{3}}\right) + 2 = 0$$

$$\Rightarrow 3\left(\frac{2}{3}\right) + m\left(\sqrt{\frac{2}{3}}\right) + 2 = 0$$

$$\Rightarrow m = -4 \times \sqrt{\frac{3}{2}} = -2\sqrt{6}$$

$$\therefore m = -2\sqrt{6}$$

**Question 4.**

$\frac{2}{3}$  and 1 are the solutions of equation  $mx^2 + nx + 6 = 0$ . Find the values of m and n.

**Solution:**

For  $x = \frac{2}{3}$  and  $x = 1$  to be solutions of the given quadratic equation it should satisfy the equation

So, substituting  $x = \frac{2}{3}$  and  $x = 1$  in the given equation, we get

$$\begin{array}{l|l} m\left(\frac{2}{3}\right)^2 + n\left(\frac{2}{3}\right) + 6 = 0 & m(1)^2 + n(1) + 6 = 0 \\ \Rightarrow m\left(\frac{4}{9}\right) + n\left(\frac{2}{3}\right) + 6 = 0 & \Rightarrow m + n + 6 = 0 \\ \Rightarrow 4m + 6n + 54 = 0 \dots(1) & \Rightarrow m + n + 6 = 0 \dots(2) \end{array}$$

Solving equations (1) and (2) simultaneously,

$$4m + 6n + 54 = 0 \dots(1)$$

$$m + n + 6 = 0 \dots(2)$$

$$(1) - (2) \times 6$$

$$\Rightarrow -2m + 18 = 0$$

$$\Rightarrow m = 9$$

Substitute in (2)

$$\Rightarrow n = -15$$

**Question 5.**

If 3 and -3 are the solutions of equation  $ax^2 + bx - 9 = 0$ . Find the values of a and b.

**Solution:**

For  $x = 3$  and  $x = -3$  to be solutions of the given quadratic equation it should satisfy the equation

So, substituting  $x = 3$  and  $x = -3$  in the given equation, we get

$$\begin{array}{l|l} a(3)^2 + b(3) - 9 = 0 & a(-3)^2 + b(-3) - 9 = 0 \\ \Rightarrow a(9) + b(3) - 9 = 0 & \Rightarrow a(9) - b(3) - 9 = 0 \\ \Rightarrow 9a + 3b - 9 = 0 \dots(1) & \Rightarrow 9a - 3b - 9 = 0 \dots(2) \end{array}$$

Solving equations (1) and (2) simultaneously,

$$9a + 3b - 9 = 0 \dots(1)$$

$$9a - 3b - 9 = 0 \dots(2)$$

$$(1) + (2)$$

$$\Rightarrow 18a - 18 = 0$$

$$\Rightarrow a = 1$$

Substitute in (2)

$$\Rightarrow b = 0$$

## Exercise 5B

### Question 1.

Without solving, comment upon the nature of roots of each of the following equations :

(i)  $7x^2 - 9x + 2 = 0$

(ii)  $6x^2 - 13x + 4 = 0$

(iii)  $25x^2 - 10x + 1 = 0$

(iv)  $x^2 + 2\sqrt{3}x - 9 = 0$

(v)  $x^2 - ax - b^2 = 0$

(vi)  $2x^2 + 8x + 9 = 0$

### Solution:

(i)  $7x^2 - 9x + 2 = 0$

$a = 7$ ,  $b = -9$  and  $c = 2$

$\therefore$  Discriminant  $= b^2 - 4ac$

$$= (-9)^2 - 4(7)(2)$$

$$= 81 - 56 = 25$$

Since  $D > 0$ , then equation has two real and unequal roots.

(ii)  $6x^2 - 13x + 4 = 0$

$a = 6$ ,  $b = -13$  and  $c = 4$

$\therefore$  Discriminant  $= b^2 - 4ac$

$$= (-13)^2 - 4(6)(4)$$

$$= 169 - 96 = 73$$

Since 73 is not a perfect square, roots are irrational

Since  $D > 0$ , then equation has two irrational and unequal roots.

(iii)  $25x^2 - 10x + 1 = 0$

$a = 25$ ,  $b = -10$  and  $c = 1$

$\therefore$  Discriminant  $= b^2 - 4ac$

$$\begin{aligned}
 &= (-10)^2 - 4(25)(1) \\
 &= 100 - 100 = 0
 \end{aligned}$$

Since  $D=0$ , then equation has two real and equal roots.

$$(iv) x^2 + 2\sqrt{3}x - 9 = 0$$

$$a = 1, b = 2\sqrt{3} \text{ and } c = -9$$

$$\therefore \text{Discriminant} = b^2 - 4ac$$

$$\begin{aligned}
 &= (2\sqrt{3})^2 - 4(1)(-9) \\
 &= 12 + 36 = 48
 \end{aligned}$$

Since 48 is not a perfect square, roots are irrational

Since  $D > 0$ , then equation has two irrational and unequal roots.

$$(v) x^2 - ax - b^2 = 0$$

$$A = 1, B = -a \text{ and } C = -b^2$$

$$\therefore \text{Discriminant} = B^2 - 4AC$$

$$\begin{aligned}
 &= (-a)^2 - 4(1)(-b^2) \\
 &= a^2 + 4b^2 = \text{a positive value}
 \end{aligned}$$

Since  $a^2 + 4b^2$  is not a perfect square, roots are irrational.

Since  $D > 0$ , then equation has two irrational and unequal roots.

$$(vi) 2x^2 + 8x + 9 = 0$$

$$a = 2, b = 8 \text{ and } c = 9$$

$$\therefore \text{Discriminant} = b^2 - 4ac$$

$$\begin{aligned}
 &= (8)^2 - 4(2)(9) \\
 &= 64 - 72 = -18 = \text{a negative value}
 \end{aligned}$$

Since  $D < 0$ , then equation has no real roots.

### Question 2.

Find the value of  $p$ , if the following quadratic equation has equal roots :  $4x^2 - (p - 2)x + 1 = 0$

**Solution:**

$$4x^2 - (p - 2)x + 1 = 0$$

Here  $a = 4$ ,  $b = -(p - 2)$  and  $c = 1$

Given: equation has equal roots

Then  $D = 0$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow [-(p - 2)]^2 - 4(4)(1) = 0$$

$$\Rightarrow p^2 + 4 - 4p - 16 = 0$$

$$\Rightarrow p^2 - 4p - 12 = 0$$

$$\Rightarrow p^2 - 6p + 2p - 12 = 0$$

$$\Rightarrow p(p - 6) + 2(p - 6) = 0$$

$$\Rightarrow (p - 6)(p + 2) = 0$$

then  $p - 6 = 0$  or  $p + 2 = 0$

$$\Rightarrow p = 6 \quad \text{or} \quad p = -2$$

### Question 3.

Find the value of 'p', if the following quadratic equations have equal roots :  $x^2 + (p - 3)x + p = 0$

#### Solution:

$$x^2 + (p - 3)x + p = 0$$

Here,  $a = 1$ ,  $b = (p - 3)$ ,  $c = p$

Since, the roots are equal,

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow (p - 3)^2 - 4(1)(p) = 0$$

$$\Rightarrow p^2 + 9 - 6p - 4p = 0$$

$$\Rightarrow p^2 - 10p + 9 = 0$$

$$\Rightarrow p^2 - 9p - p + 9 = 0$$

$$\Rightarrow p(p - 9) - 1(p - 9) = 0$$

$$\Rightarrow (p - 9)(p - 1) = 0$$

$$\Rightarrow p - 9 = 0 \quad \text{or} \quad p - 1 = 0$$

$$\Rightarrow p = 9 \quad \text{or} \quad p = 1$$

### Question 4.

The equation  $3x^2 - 12x + (n - 5) = 0$  has equal roots. Find the value of n.

**Solution:**

$$3x^2 - 12x + (n - 5) = 0$$

Here  $a = 3$ ,  $b = -12$  and  $c = n - 5$

Given: equation has equal roots

Then  $D = 0$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow [-12]^2 - 4(3)(n - 5) = 0$$

$$\Rightarrow 144 - 12n + 60 = 0$$

$$\Rightarrow -12n = -204$$

$$\Rightarrow n = \frac{-204}{-12} = 17$$

**Question 5.**

Find the value of  $m$ , if the following equation has equal roots :  $(m - 2)x^2 - (5 + m)x + 16 = 0$

**Solution:**

$$(m - 2)x^2 - (5 + m)x + 16 = 0$$

Here  $a = m - 2$ ,  $b = -(5 + m)$  and  $c = 16$

Given: equation has equal roots

Then  $D = 0$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow [-(5 + m)]^2 - 4(m - 2)(16) = 0$$

$$\Rightarrow 25 + m^2 + 10m - 64m + 128 = 0$$

$$\Rightarrow m^2 - 54m + 153 = 0$$

$$\Rightarrow m^2 - 51m - 3m + 153 = 0$$

$$\Rightarrow m(m - 51) - 3(m - 51) = 0$$

$$\Rightarrow (m - 51)(m - 3) = 0$$

$$\text{then } m - 51 = 0 \quad \text{or} \quad m - 3 = 0$$

$$\Rightarrow m = 51 \quad \text{or} \quad m = 3$$

**Question 6.**

Find the value of  $p$  for which the equation  $3x^2 - 6x + k = 0$  has distinct and real roots.

**Solution:**

$$3x^2 - 6x + k = 0$$

Here,  $a = 3$ ,  $b = -6$  and  $c = k$

Since the roots are distinct and real,

$$b^2 - 4ac > 0$$

$$\Rightarrow (-6)^2 - 4 \times 3 \times k > 0$$

$$\Rightarrow 36 - 12k > 0$$

$$\Rightarrow 36 > 12k$$

$$\Rightarrow 3 > k$$

$$\Rightarrow k < 3$$

**Exercise 5C****Question 1.**

Solve :  $x^2 - 10x - 24 = 0$

**Solution:**

$$x^2 - 10x - 24 = 0$$

$$\Rightarrow x^2 - 12x + 2x - 24 = 0$$

$$\Rightarrow x(x - 12) + 2(x - 12) = 0$$

$$\Rightarrow (x - 12)(x + 2) = 0$$

since  $x - 12 = 0$  or  $x + 2 = 0$

then  $x = 12$  or  $x = -2$

**Question 2.**

Solve :  $x^2 - 16 = 0$

**Solution:**

$$x^2 - 16 = 0$$

$$\Rightarrow x^2 - 4^2 = 0$$

$$\Rightarrow (x + 4)(x - 4) = 0$$

Since  $x + 4 = 0$  or  $x - 4 = 0$

then  $x = -4$  or  $x = 4$

**Question 3.**

Solve:  $2x^2 - \frac{1}{2}x = 0$

**Solution:**

$$2x^2 - \frac{1}{2}x = 0$$

$$\Rightarrow x(2x - \frac{1}{2}) = 0$$

since  $x=0$  or  $2x - \frac{1}{2} = 0$

then  $x=0$  or  $x = \frac{1}{4}$

**Question 4.**

Solve :  $x(x - 5) = 24$

**Solution:**

$$x(x-5)=24$$

$$\Rightarrow x^2 - 5x - 24 = 0$$

$$\Rightarrow x^2 - 8x + 3x - 24 = 0$$

$$\Rightarrow x(x-8) + 3(x-8) = 0$$

$$\Rightarrow (x-8)(x+3) = 0$$

Since  $x-8=0$  or  $x+3=0$

then  $x=8$  or  $x = -3$

**Question 5.**

Solve:  $\frac{9}{2}x = 5 + x^2$

**Solution:**

$$\frac{9}{2}x = 5 + x^2$$

$$\Rightarrow 9x = 10 + 2x^2$$

$$\Rightarrow 2x^2 - 9x + 10 = 0$$

$$\Rightarrow 2x^2 - 5x - 4x + 10 = 0$$

$$\Rightarrow x(2x-5) - 2(2x-5) = 0$$

$$\Rightarrow (2x-5)(x-2) = 0$$

$$\text{Since } 2x-5=0 \quad \text{or} \quad x-2=0$$

$$\text{then } x = \frac{5}{2} \quad \text{or} \quad x = 2$$

**Question 6.**

$$\text{Solve: } \frac{6}{x} = 1 + x$$

**Solution:**

$$\frac{6}{x} = 1 + x$$

$$\Rightarrow 6 = x + x^2$$

$$\Rightarrow x^2 + x - 6 = 0$$

$$\Rightarrow x^2 + 3x - 2x - 6 = 0$$

$$\Rightarrow x(x+3) - 2(x+3) = 0$$

$$\Rightarrow (x+3)(x-2) = 0$$

$$\text{since } x+3=0 \quad \text{or} \quad x-2=0$$

$$\text{then } x = -3 \quad \text{or} \quad x = 2$$

**Question 7.**

$$\text{Solve: } x = \frac{3x+1}{4x}$$

**Solution:**

$$x = \frac{3x+1}{4x}$$

$$\Rightarrow 4x^2 = 3x + 1$$

$$\Rightarrow 4x^2 - 3x - 1 = 0$$

$$\Rightarrow 4x^2 - 4x + x - 1 = 0$$

$$\Rightarrow 4x(x-1) + 1(x-1) = 0$$

$$\Rightarrow (x-1)(4x+1)=0$$

Since  $x-1=0$  or  $4x+1=0$

then  $x=1$  or  $x=\frac{-1}{4}$

**Question 8.**

Solve:  $x + \frac{1}{x} = 2.5$

**Solution:**

$$x + \frac{1}{x} = 2.5$$

$$\Rightarrow \frac{x^2 + 1}{x} = \frac{5}{2}$$

$$\Rightarrow 2x^2 + 2 = 5x$$

$$\Rightarrow 2x^2 - 5x + 2 = 0$$

$$\Rightarrow 2x^2 - 4x - x + 2 = 0$$

$$\Rightarrow 2x(x-2) - 1(x-2) = 0$$

$$\Rightarrow (x-2)(2x-1) = 0$$

Since  $x-2=0$  or  $2x-1=0$

then  $x=2$  or  $x=\frac{1}{2}$

**Question 9.**

Solve:  $(2x - 3)^2 = 49$

**Solution:**

$$(2x-3)^2 = 49$$

Taking square root on both sides

$$2x-3 = \pm 7$$

When  $2x-3=7 \Rightarrow 2x=10 \Rightarrow x=5$

and, when  $2x-3=-7 \Rightarrow 2x=-4 \Rightarrow x=-2$

**Question 10.**

Solve :  $2(x^2 - 6) = 3(x - 4)$

**Solution:**

$$2(x^2 - 6) = 3(x - 4)$$

$$\Rightarrow 2x^2 - 12 = 3x - 12$$

$$\Rightarrow 2x^2 - 3x = 0$$

$$\Rightarrow x(2x - 3) = 0$$

since  $x=0$  or  $2x-3=0$

then  $x=0$  or  $x=\frac{3}{2}$

**Question 11.**

Solve :  $(x + 1)(2x + 8) = (x + 7)(x + 3)$

**Solution:**

$$(x+1)(2x+8)=(x+7)(x+3)$$

$$\Rightarrow 2x^2 + 8x + 2x + 8 = x^2 + 3x + 7x + 21$$

$$\Rightarrow 2x^2 + 10x + 8 = x^2 + 10x + 21$$

$$\Rightarrow x^2 - 13 = 0$$

$$\Rightarrow x^2 - (\sqrt{13})^2 = 0$$

$$\Rightarrow (x + \sqrt{13})(x - \sqrt{13}) = 0$$

If  $x + \sqrt{13} = 0$  or  $x - \sqrt{13} = 0$

$$\Rightarrow x = -\sqrt{13} \quad \text{or} \quad x = \sqrt{13}$$

**Question 12.**

Solve :  $x^2 - (a + b)x + ab = 0$

**Solution:**

$$x^2 - (a + b)x + ab = 0$$

$$\Rightarrow x^2 - ax - bx + ab = 0$$

$$\Rightarrow x(x - a) - b(x - a) = 0$$

$$\Rightarrow (x - a)(x - b) = 0$$

$$\text{since } x - a = 0 \quad \text{or} \quad x - b = 0$$

$$\text{then } x = a \quad \text{or} \quad x = b$$

**Question 13.**

$$(x + 3)^2 - 4(x + 3) - 5 = 0$$

**Solution:**

$$(x + 3)^2 - 4(x + 3) - 5 = 0$$

$$\text{Let } x + 3 = y$$

$$\text{then } y^2 - 4y - 5 = 0$$

$$\Rightarrow y^2 - 5y + y - 5 = 0$$

$$\Rightarrow y(y - 5) + 1(y - 5) = 0$$

$$\Rightarrow (y - 5)(y + 1) = 0$$

$$\text{If } y - 5 = 0 \quad \text{or} \quad y + 1 = 0$$

$$\text{then } y = 5 \quad \text{or} \quad y = -1$$

$$\Rightarrow x + 3 = 5 \quad \text{or} \quad x + 3 = -1$$

$$\Rightarrow x = 2 \quad \text{or} \quad x = -4$$

**Question 14.**

$$4(2x - 3)^2 - (2x - 3) - 14 = 0$$

**Solution:**

$$4(2x - 3)^2 - (2x - 3) - 14 = 0$$

$$\text{Let } 2x - 3 = y$$

$$\text{then } 4y^2 - y - 14 = 0$$

$$\Rightarrow 4y^2 - 8y + 7y - 14 = 0$$

$$\Rightarrow 4y(y - 2) + 7(y - 2) = 0$$

$$\Rightarrow (y - 2)(4y + 7) = 0$$

$$\text{If } y - 2 = 0 \quad \text{or} \quad 4y + 7 = 0$$

$$\Rightarrow y = 2 \quad \text{or} \quad y = \frac{-7}{4}$$

$$\Rightarrow 2x - 3 = 2 \quad \text{or} \quad 2x - 3 = \frac{-7}{4}$$

$$\Rightarrow 2x = 5 \quad \text{or} \quad 2x = \frac{5}{4}$$

$$\Rightarrow x = \frac{5}{2} \quad \text{or} \quad x = \frac{5}{8}$$

**Question 15.**

$$\text{Solve: } \frac{3x - 2}{2x - 3} = \frac{3x - 8}{x + 4}$$

**Solution:**

$$\frac{3x - 2}{2x - 3} = \frac{3x - 8}{x + 4}$$

$$\Rightarrow (3x - 2)(x + 4) = (2x - 3)(3x - 8)$$

$$\Rightarrow 3x^2 + 12x - 2x - 8 = 6x^2 - 16x - 9x + 24$$

$$\Rightarrow 3x^2 + 10x - 8 = 6x^2 - 25x + 24$$

$$\Rightarrow 3x^2 - 35x + 32 = 0$$

$$\Rightarrow 3x^2 - 32x - 3x + 32 = 0$$

$$\Rightarrow x(3x - 32) - 1(3x - 32) = 0$$

$$\Rightarrow (x - 1)(3x - 32) = 0$$

$$\text{If } x - 1 = 0 \quad \text{or} \quad 3x - 32 = 0$$

$$\Rightarrow x = 1 \quad \text{or} \quad x = \frac{32}{3} = 10\frac{2}{3}$$

**Question 16.**

$2x^2 - 9x + 10 = 0$ , When

(i)  $x \in \mathbb{N}$

(ii)  $x \in \mathbb{Q}$

**Solution:**

$$2x^2 - 9x + 10 = 0$$

$$\Rightarrow 2x^2 - 5x - 4x + 10 = 0$$

$$\Rightarrow x(2x - 5) - 2(2x - 5) = 0$$

$$\Rightarrow (2x - 5)(x - 2) = 0$$

$$\Rightarrow 2x - 5 = 0 \text{ or } x - 2 = 0$$

$$\Rightarrow x = \frac{5}{2} \text{ or } x = 2$$

(i) When  $x \in \mathbb{N}$ , we have  $x = 2$

(ii) When  $x \in \mathbb{Q}$ , we have  $x = 2, \frac{5}{2}$

**Question 17.**

$$\text{Solve: } \frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2}$$

**Solution:**

$$\frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2}$$

$$\Rightarrow \frac{(x-3)^2 + (x+3)^2}{(x+3)(x-3)} = \frac{5}{2}$$

$$\Rightarrow \frac{x^2 - 6x + 9 + x^2 + 6x + 9}{x^2 - 9} = \frac{5}{2}$$

$$\Rightarrow 2(2x^2 + 18) = 5(x^2 - 9)$$

$$\Rightarrow 4x^2 + 36 = 5x^2 - 45$$

$$\Rightarrow x^2 - 81 = 0$$

$$\Rightarrow x^2 - 9^2 = 0$$

$$\Rightarrow (x+9)(x-9) = 0$$

$$\text{If } x+9=0 \text{ or } x-9=0$$

$$\text{then } x=-9 \text{ or } x=9$$

**Question 18.**

$$\text{Solve: } \frac{4}{x+2} - \frac{1}{x+3} = \frac{4}{2x+1}$$

**Solution:**

$$\begin{aligned} \frac{4}{x+2} - \frac{1}{x+3} &= \frac{4}{2x+1} \\ \Rightarrow \frac{4(x+3) - 1(x+2)}{(x+2)(x+3)} &= \frac{4}{2x+1} \\ \Rightarrow \frac{4x+12 - x - 2}{x^2 + 2x + 3x + 6} &= \frac{4}{2x+1} \\ \Rightarrow \frac{3x+10}{x^2 + 5x + 6} &= \frac{4}{2x+1} \\ \Rightarrow (3x+10)(2x+1) &= 4(x^2 + 5x + 6) \\ \Rightarrow 6x^2 + 3x + 20x + 10 &= 4x^2 + 20x + 24 \\ \Rightarrow 2x^2 + 3x - 14 &= 0 \\ \Rightarrow 2x^2 + 7x - 4x - 14 &= 0 \\ \Rightarrow x(2x+7) - 2(2x+7) &= 0 \\ \Rightarrow (2x+7)(x-2) &= 0 \\ \text{If } 2x+7=0 \quad \text{or} \quad x-2 &= 0 \\ \text{then } x &= \frac{-7}{2} \quad \text{or} \quad x=2 \end{aligned}$$

**Question 19.**

$$\text{Solve: } \frac{5}{x-2} - \frac{3}{x+6} = \frac{4}{x}$$

**Solution:**

$$\begin{aligned} \frac{5}{x-2} - \frac{3}{x+6} &= \frac{4}{x} \\ \Rightarrow \frac{5(x+6) - 3(x-2)}{(x-2)(x+6)} &= \frac{4}{x} \\ \Rightarrow \frac{5x+30 - 3x+6}{x^2 + 6x - 2x - 12} &= \frac{4}{x} \end{aligned}$$

$$\begin{aligned}
\Rightarrow \frac{2x+36}{x^2+4x-12} &= \frac{4}{x} \\
\Rightarrow 4x^2+16x-48 &= 2x^2+36x \\
\Rightarrow 2x^2-20x-48 &= 0 \\
\Rightarrow x^2-10x-24 &= 0 \\
\Rightarrow x^2-12x+2x-24 &= 0 \\
\Rightarrow x(x-12)+2(x-12) &= 0 \\
\Rightarrow (x-12)(x+2) &= 0 \\
\text{If } x-12=0 &\text{ or } x+2=0 \\
\text{then } x=12 &\text{ or } x=-2
\end{aligned}$$

**Question 20.**

$$\text{Solve: } \left(1 + \frac{1}{x+1}\right) \left(1 - \frac{1}{x-1}\right) = \frac{7}{8}$$

**Solution:**

$$\begin{aligned}
\left(1 + \frac{1}{x+1}\right) \left(1 - \frac{1}{x-1}\right) &= \frac{7}{8} \\
\Rightarrow \left(\frac{x+1+1}{x+1}\right) \left(\frac{x-1-1}{x-1}\right) &= \frac{7}{8} \\
\Rightarrow \left(\frac{x+2}{x+1}\right) \left(\frac{x-2}{x-1}\right) &= \frac{7}{8} \\
\Rightarrow \frac{x^2-4}{x^2-1} &= \frac{7}{8} \\
\Rightarrow 8x^2-32 &= 7x^2-7 \\
\Rightarrow x^2 &= 25 \\
\Rightarrow x &= \pm 5
\end{aligned}$$

**Question 21.**

Find the quadratic equation, whose solution set is :

(i) {3, 5} (ii) {-2, 3}

**Solution:**(i) Since solution set is  $\{3, 5\}$ 

$$\Rightarrow x=3 \text{ or } x=5$$

$$\Rightarrow x-3=0 \text{ or } x-5=0$$

$$\Rightarrow (x-3)(x-5)=0$$

$$\Rightarrow x^2 - 5x - 3x + 15 = 0$$

$$\Rightarrow x^2 - 8x + 15 = 0 \text{ which is the required equation.}$$

(ii) Since solution set is  $\{-2, 3\}$ 

$$\Rightarrow x=-2 \text{ or } x=3$$

$$\Rightarrow x+2=0 \text{ or } x-3=0$$

$$\Rightarrow (x+2)(x-3)=0$$

$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow x^2 - x - 6 = 0 \text{ which is the required equation.}$$

(iii) Since solution set is  $\{5, -4\}$ 

$$\Rightarrow x=5 \text{ or } x=-4$$

$$\Rightarrow x-5=0 \text{ or } x+4=0$$

$$\Rightarrow (x-5)(x+4)=0$$

$$\Rightarrow x^2 - 5x + 4x - 20 = 0$$

$$\Rightarrow x^2 - x - 20 = 0 \text{ which is the required equation.}$$

(iv) Since solution set is  $\left\{-3, \frac{-2}{5}\right\}$ 

$$\Rightarrow x=-3 \text{ or } x=\frac{-2}{5}$$

$$\Rightarrow x+3=0 \text{ or } 5x+2=0$$

$$\Rightarrow (x+3)(5x+2)=0$$

$$\Rightarrow 5x^2 + 2x + 15x + 6 = 0$$

$$\Rightarrow 5x^2 + 17x + 6 = 0 \text{ which is the required equation.}$$

**Question 22.**

$$\text{Solve : } \frac{x}{3} + \frac{3}{6-x} = \frac{2(6+x)}{15}; (x \neq 6)$$

**Solution:**

$$\begin{aligned} \frac{x}{3} + \frac{3}{6-x} &= \frac{2(6+x)}{15} \\ \Rightarrow \frac{x(6-x) + 3 \times 3}{3(6-x)} &= \frac{12+2x}{15} \\ \Rightarrow \frac{x(6-x) + 3 \times 3}{6-x} &= \frac{12+2x}{5} \\ \Rightarrow \frac{6x - x^2 + 9}{6-x} &= \frac{12+2x}{5} \\ \Rightarrow 30x - 5x^2 + 45 &= 72 + 12x - 12x - 2x^2 \\ \Rightarrow 30x - 5x^2 + 45 &= 72 - 2x^2 \\ \Rightarrow 3x^2 - 30x + 27 &= 0 \\ \Rightarrow x^2 - 10x + 9 &= 0 \\ \Rightarrow x^2 - 9x - x + 9 &= 0 \\ \Rightarrow x(x-9) - 1(x-9) &= 0 \\ \Rightarrow (x-9)(x-1) &= 0 \\ \Rightarrow x-9=0 \text{ or } x-1 &= 0 \\ \Rightarrow x=9 \text{ or } x=1 \end{aligned}$$

**Question 23.**

Solve the equation  $9x^2 + \frac{3x}{4} + 2 = 0$ , if possible, for real values of  $x$ .

**Solution:**

$$\begin{aligned} 9x^2 + \frac{3x}{4} + 2 &= 0 \\ \Rightarrow \frac{36x^2 + 3x + 8}{4} &= 0 \\ \Rightarrow 36x^2 + 3x + 8 &= 0 \\ \text{Here, } a &= 36, b = 3 \text{ and } c = 8 \\ \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$

$$\begin{aligned}
 &= \frac{-3 \pm \sqrt{(3)^2 - 4 \times 36 \times 8}}{2 \times 36} \\
 &= \frac{-3 \pm \sqrt{9 - 1152}}{72} \\
 &= \frac{-3 \pm \sqrt{-1143}}{72}
 \end{aligned}$$

Since  $\sqrt{-1143}$  is not possible, we cannot solve the given equation for  $x$ .

**Question 24.**

Find the value of  $x$ , if  $a + 1 = 0$  and  $x^2 + ax - 6 = 0$ .

**Solution:**

If  $a + 1 = 0$ , then  $a = -1$

Put this value in the given equation  $x^2 + ax - 6 = 0$

$$x^2 - x - 6 = 0$$

$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow x(x - 3) + 2(x - 3) = 0$$

$$\Rightarrow (x - 3)(x + 2) = 0$$

$$\text{If } x - 3 = 0 \quad \text{or} \quad x + 2 = 0$$

$$\text{then } x = 3 \quad \text{or} \quad x = -2$$

**Question 25.**

Find the value of  $x$ , if  $a + 7 = 0$ ;  $b + 10 = 0$  and  $12x^2 = ax - b$ .

**Solution:**

If  $a + 7 = 0$ , then  $a = -7$

and  $b + 10 = 0$ , then  $b = -10$

Put these values of  $a$  and  $b$  in the given equation

$$12x^2 = (-7)x - (-10)$$

$$\Rightarrow 12x^2 + 7x - 10 = 0$$

$$\Rightarrow 12x^2 + 15x - 8x - 10 = 0$$

$$\Rightarrow 3x(4x + 5) - 2(4x + 5) = 0$$

$$\Rightarrow (4x + 5)(3x - 2) = 0$$

$$\text{If } 4x + 5 = 0 \quad \text{or} \quad 3x - 2 = 0$$

$$\text{then } x = \frac{-5}{4} \quad \text{or} \quad x = \frac{2}{3}$$

**Question 26.**

Use the substitution  $y = 2x + 3$  to solve for  $x$ , if  $4(2x+3)^2 - (2x+3) - 14 = 0$ .

**Solution:**

$$4(2x+3)^2 - (2x+3) - 14 = 0$$

$$\text{Put } 2x+3 = y$$

$$4y^2 - y - 14 = 0$$

$$\Rightarrow 4y^2 - 8y + 7y - 14 = 0$$

$$\Rightarrow 4y(y - 2) + 7(y - 2) = 0$$

$$\Rightarrow (y - 2)(4y + 7) = 0$$

$$\text{If } y - 2 = 0 \quad \text{or} \quad 4y + 7 = 0$$

$$\text{then } 2x + 3 - 2 = 0 \quad \text{or} \quad 4(2x + 3) + 7 = 0$$

$$\Rightarrow 2x = -1 \quad \text{or} \quad 8x = -19$$

$$\Rightarrow x = \frac{-1}{2} \quad \text{or} \quad x = \frac{-19}{8}$$

**Question 27.**

Without solving the quadratic equation  $6x^2 - x - 2 = 0$ , find whether  $x = 2/3$  is a solution of this equation or not.

**Solution:**

Consider the equation,  $6x^2 - x - 2 = 0$

Put  $x = \frac{2}{3}$  in L.H.S.

$$\text{L.H.S.} = 6\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right) - 2$$

$$= \frac{24}{9} - \frac{2}{3} - 2$$

$$= \frac{24 - 6 - 18}{9} = 0 = \text{R.H.S.}$$

Since L.H.S. = R.H.S., then  $x = \frac{2}{3}$  is a solution of the given equation.

**Question 28.**

Determine whether  $x = -1$  is a root of the equation  $x^2 - 3x + 2 = 0$  or not.

**Solution:**

$$x^2 - 3x + 2 = 0$$

Put  $x = -1$  in L.H.S.

$$\text{L.H.S.} = (-1)^2 - 3(-1) + 2$$

$$= 1 + 3 + 2 = 6 \neq \text{R.H.S.}$$

Then  $x = -1$  is not the solution of the given equation.

**Question 29.**

If  $x = \frac{2}{3}$  is a solution of the quadratic equation  $7x^2 + mx - 3 = 0$ ; Find the value of  $m$ .

**Solution:**

$$7x^2 + mx - 3 = 0$$

Given  $x = \frac{2}{3}$  is the solution of the given equation.

Put given value of  $x$  in the given equation

$$7\left(\frac{2}{3}\right)^2 + m\left(\frac{2}{3}\right) - 3 = 0$$

$$\Rightarrow \frac{28}{9} + \frac{2m}{3} - 3 = 0$$

$$\Rightarrow 28 + 6m - 27 = 0$$

$$\Rightarrow 6m = -1$$

$$\Rightarrow m = \frac{-1}{6}$$

**Question 30.**

If  $x = -3$  and  $x = \frac{2}{3}$  are solutions of quadratic equation  $mx^2 + 7x + n = 0$ , find the values of  $m$  and  $n$ .

**Solution:**

$$mx^2 + 7x + n = 0$$

Put  $x = -3$  in given equation

$$m(-3)^2 + 7(-3) + n = 0$$

$$\Rightarrow 9m - 21 + n = 0$$

$$\Rightarrow 9m + n = 21 \text{ --- (1)}$$

Put  $x = \frac{2}{3}$  in given equation

$$m\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + n = 0$$

$$\Rightarrow \frac{4m}{9} + \frac{14}{3} + n = 0$$

$$\Rightarrow 4m + 9n = -42 \text{ --- (2)}$$

solving these equations we get

$$m = 3 \text{ and } n = -6$$

**Question 31.**

If quadratic equation  $x^2 - (m + 1)x + 6 = 0$  has one root as  $x = 3$ ; find the value of  $m$  and the root of the equation.

**Solution:**

$$x^2 - (m + 1)x + 6 = 0$$

Put  $x = 3$  in the given equation

$$(3)^2 - (m + 1)(3) + 6 = 0$$

$$\Rightarrow 9 - 3m - 3 + 6 = 0$$

$$\Rightarrow -3m = -12$$

$$\Rightarrow m = 4$$

Put this value of  $m$  in the given equation, we get

$$x^2 - 5x + 6 = 0$$

$$\Rightarrow x^2 - 3x - 2x + 6 = 0$$

$$\Rightarrow x(x - 3) - 2(x - 3) = 0$$

$$\Rightarrow (x - 3)(x - 2) = 0$$

$$\text{If } x - 3 = 0 \quad \text{or} \quad x - 2 = 0$$

$$\text{then } x = 3 \quad \text{or} \quad x = 2$$

$\therefore 2$  is the other root of the given equation.

**Question 32.**

Given that 2 is a root of the equation  $3x^2 - p(x + 1) = 0$  and that the equation  $px^2 - qx + 9 = 0$  has equal roots, find the values of  $p$  and  $q$ .

**Solution:**

Since 2 is a root of the equation  $3x^2 - p(x + 1) = 0$

$$\Rightarrow 3(2)^2 - p(2 + 1) = 0$$

$$\Rightarrow 3 \times 4 - 3p = 0$$

$$\Rightarrow 12 - 3p = 0$$

$$\Rightarrow 3p = 12$$

$$\Rightarrow p = 4$$

Now, the other equation becomes  $4x^2 - qx + 9 = 0$

Here,  $a = 4$ ,  $b = -q$  and  $c = 9$

Since the roots are equal, we have

$$b^2 - 4ac = 0$$

$$\Rightarrow (-q)^2 - 4 \times 4 \times 9 = 0$$

$$\Rightarrow q^2 - 144 = 0$$

$$\Rightarrow q^2 = 144$$

$$\Rightarrow q = 12$$

Hence,  $p = 4$  and  $q = 12$ .

**Question 33.**

Solve :  $\frac{x}{a} - \frac{a+b}{x} = \frac{b(a+b)}{ax}$

**Solution:**

$$\frac{x}{a} - \frac{a+b}{x} = \frac{b(a+b)}{ax}$$

$$\Rightarrow \frac{x^2 - a^2 - ab}{ax} = \frac{ab + b^2}{ax}$$

$$\Rightarrow x^2 - a^2 - ab = ab + b^2$$

$$\Rightarrow x^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow x^2 = (a+b)^2$$

$$\Rightarrow x = a+b$$

**Question 34.**

$$\text{Solve : } \left( \frac{1200}{x} + 2 \right) (x - 10) - 1200 = 60$$

**Solution:**

$$\left( \frac{1200}{x} + 2 \right) (x - 10) - 1200 = 60$$

$$\Rightarrow 2 \left( \frac{600}{x} + 1 \right) (x - 10) = 1260$$

$$\Rightarrow \left( \frac{600}{x} + 1 \right) (x - 10) = 630$$

$$\Rightarrow \left( \frac{600 + x}{x} \right) (x - 10) = 630$$

$$\Rightarrow 600x - 6000 + x^2 - 10x = 630x$$

$$\Rightarrow x^2 - 40x - 6000 = 0$$

$$\Rightarrow x^2 - 100x + 60x - 6000 = 0$$

$$\Rightarrow x(x - 100) + 60(x - 100) = 0$$

$$\Rightarrow (x - 100)(x + 60) = 0$$

$$\Rightarrow x - 100 = 0 \text{ or } x + 60 = 0$$

$$\Rightarrow x = 100 \text{ or } x = -60$$

**Question 35.**

If -1 and 3 are the roots of  $x^2 + px + q = 0$ , find the values of p and q.

**Solution:**

Since -1 is a root of  $x^2 + px + q = 0$ , we have

$$(-1)^2 + p(-1) + q = 0$$

$$\Rightarrow 1 - p + q = 0$$

$$\Rightarrow -p + q = -1 \quad \dots \text{(i)}$$

Also, 3 is a root of  $x^2 + px + q = 0$ .

$$\Rightarrow (3)^2 + p(3) + q = 0$$

$$\Rightarrow 9 + 3p + q = 0$$

$$\Rightarrow 3p + q = -9 \quad \dots \text{(ii)}$$

Subtracting equation (ii) from (i), we get

$$-4p = 8$$

$$\Rightarrow p = -2$$

$$\Rightarrow -(-2) + q = -1 \quad \dots [\text{From (i)}]$$

$$\Rightarrow 2 + q = -1$$

$$\Rightarrow q = -3$$

Hence,  $p = -2$  and  $q = -3$ .

## Exercise 5D

### Question 1.

Solve each of the following equations using the formula :

$$(i) x^2 - 6x = 27 \quad (ii) x^2 - 10x + 21 = 0$$

$$(iii) x^2 + 6x - 10 = 0 \quad (iv) x^2 + 2x - 6 = 0$$

$$(v) 3x^2 + 2x - 1 = 0 \quad (vi) 2x^2 + 7x + 5 = 0$$

$$(vii) \frac{2}{3}x = -\frac{1}{6}x^2 - \frac{1}{3} \quad (viii) \frac{1}{15}x^2 + \frac{5}{3} = \frac{2}{3}x$$

$$(ix) x^2 - 6 = 2\sqrt{2}x \quad (x) \frac{4}{x} - 3 = \frac{5}{2x+3}$$

$$(xi) \frac{2x+3}{x+3} = \frac{x+4}{x+2} \quad (xii) \sqrt{6}x^2 - 4x - 2\sqrt{6} = 0$$

$$(xiii) \frac{2x}{x-4} + \frac{2x-5}{x-3} = 8\frac{1}{3} \quad (xiv) \frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$$

### Solution:

$$(i) x^2 - 6x = 27$$

$$\Rightarrow x^2 - 6x - 27 = 0$$

Here  $a=1$ ,  $b=-6$  and  $c=-27$

$$\text{Then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-27)}}{2(1)}$$

$$= \frac{6 \pm 12}{2} = \frac{6+12}{2} \text{ and } \frac{6-12}{2} = 9 \text{ and } -3$$

$$(ii) x^2 - 10x + 21 = 0$$

Here  $a=1$ ,  $b=-10$  and  $c=21$

$$\begin{aligned} \text{Then } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(21)}}{2(1)} \\ &= \frac{10 \pm 4}{2} = \frac{10+4}{2} \text{ and } \frac{10-4}{2} = 7 \text{ and } 3 \end{aligned}$$

$$(iii) x^2 + 6x - 10 = 0$$

Here  $a=1$ ,  $b=6$  and  $c=-10$

$$\begin{aligned} \text{Then } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-10)}}{2(1)} \\ &= \frac{-6 \pm \sqrt{76}}{2} = \frac{-6 + 2\sqrt{19}}{2} \text{ and } \frac{-6 - 2\sqrt{19}}{2} = -3 + \sqrt{19} \text{ and } -3 - \sqrt{19} \end{aligned}$$

$$(iv) x^2 + 2x - 6 = 0$$

Here  $a=1$ ,  $b=2$  and  $c=-6$

$$\begin{aligned} \text{Then } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-6)}}{2(1)} \\ &= \frac{-2 \pm \sqrt{28}}{2} = \frac{-2 \pm 2\sqrt{7}}{2} = -1 \pm \sqrt{7} \end{aligned}$$

$$(v) 3x^2 + 2x - 1 = 0$$

Here  $a=3$ ,  $b=2$  and  $c=-1$

$$\begin{aligned} \text{Then } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(-1)}}{2(3)} \end{aligned}$$

$$= \frac{-2 \pm 4}{6} = \frac{-2+4}{6} \text{ and } \frac{-2-4}{6} = \frac{1}{3} \text{ and } -1$$

$$(vi) 2x^2 + 7x + 5 = 0$$

Here  $a=2$ ,  $b=7$  and  $c=5$

$$\text{Then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{-7 \pm 3}{4} = \frac{-7+3}{4} \text{ and } \frac{-7-3}{4} = -1 \text{ and } -\frac{5}{2}$$

$$(vii) \frac{2}{3}x = -\frac{1}{6}x^2 - \frac{1}{3}$$

$$\Rightarrow 4x = -x^2 - 2$$

$$\Rightarrow x^2 + 4x + 2 = 0$$

Here  $a=1$ ,  $b=4$  and  $c=2$

$$\text{Then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

$$(viii) \frac{1}{15}x^2 + \frac{5}{3} = \frac{2}{3}x$$

$$\Rightarrow x^2 + 25 = 10x$$

$$\Rightarrow x^2 - 10x + 25 = 0$$

Here  $a=1$ ,  $b=-10$  and  $c=25$

$$\text{Then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{0}}{2} = 5$$

$$(ix) \quad x^2 - 6 = 2\sqrt{2}x$$

$$\Rightarrow x^2 - 2\sqrt{2}x - 6 = 0$$

Here  $a=1$ ,  $b=-2\sqrt{2}$  and  $c=-6$

$$\begin{aligned} \text{Then } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(1)(-6)}}{2(1)} \\ &= \frac{2\sqrt{2} \pm \sqrt{32}}{2} = \frac{2\sqrt{2} \pm 4\sqrt{2}}{2} = \frac{2\sqrt{2} + 4\sqrt{2}}{2} \text{ and } \frac{2\sqrt{2} - 4\sqrt{2}}{2} \\ &= \frac{6\sqrt{2}}{2} \text{ and } \frac{-2\sqrt{2}}{2} = 3\sqrt{2} \text{ and } -\sqrt{2} \end{aligned}$$

$$(x) \quad \frac{4}{x} - 3 = \frac{5}{2x+3}$$

$$\Rightarrow \frac{4-3x}{x} = \frac{5}{2x+3}$$

$$\Rightarrow (4-3x)(2x+3) = 5x$$

$$\Rightarrow 8x + 12 - 6x^2 - 9x = 5x$$

$$\Rightarrow 6x^2 + 6x - 12 = 0$$

$$\Rightarrow x^2 + x - 2 = 0$$

Here  $a=1$ ,  $b=1$  and  $c=-2$

$$\begin{aligned} \text{Then } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-2)}}{2(1)} \\ &= \frac{-1 \pm \sqrt{9}}{2} = \frac{-1 \pm 3}{2} = \frac{-1+3}{2} \text{ and } \frac{-1-3}{2} = 1 \text{ and } -2 \end{aligned}$$

$$(xi) \quad \frac{2x+3}{x+3} = \frac{x+4}{x+2}$$

$$\Rightarrow (2x+3)(x+2) = (x+3)(x+4)$$

$$\Rightarrow 2x^2 + 4x + 3x + 6 = x^2 + 4x + 3x + 12$$

$$\Rightarrow x^2 - 6 = 0$$

Here  $a=1$ ,  $b=0$  and  $c=-6$

$$\begin{aligned} \text{Then } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(0) \pm \sqrt{(0)^2 - 4(1)(-6)}}{2(1)} \\ &= \frac{0 \pm \sqrt{24}}{2} = \frac{0 \pm 2\sqrt{6}}{2} = -\sqrt{6} \text{ and } \sqrt{6} \end{aligned}$$

$$(xii) \sqrt{6}x^2 - 4x - 2\sqrt{6} = 0$$

Here  $a=\sqrt{6}$ ,  $b=-4$  and  $c=-2\sqrt{6}$

$$\begin{aligned} \text{Then } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(\sqrt{6})(-2\sqrt{6})}}{2(\sqrt{6})} \\ &= \frac{4 \pm \sqrt{64}}{2\sqrt{6}} = \frac{4 \pm 8}{2\sqrt{6}} = \frac{4+8}{2\sqrt{6}} \text{ and } \frac{4-8}{2\sqrt{6}} \\ &= \frac{6}{\sqrt{6}} \text{ and } \frac{-2}{\sqrt{6}} = \sqrt{6} \text{ and } \frac{-\sqrt{6}}{3} \end{aligned}$$

$$(xiii) \frac{2x}{x-4} + \frac{2x-5}{x-3} = 8\frac{1}{3}$$

$$\Rightarrow \frac{2x(x-3) + (x-4)(2x-5)}{(x-4)(x-3)} = \frac{25}{3}$$

$$\Rightarrow \frac{2x^2 - 6x + 2x^2 - 5x - 8x + 20}{x^2 - 3x - 4x + 12} = \frac{25}{3}$$

$$\Rightarrow \frac{4x^2 - 19x + 20}{x^2 - 7x + 12} = \frac{25}{3}$$

$$\Rightarrow 25x^2 - 175x + 300 = 12x^2 - 57x + 60$$

$$\Rightarrow 13x^2 - 118x + 240 = 0$$

Here  $a=13$ ,  $b=-118$  and  $c=240$

$$\begin{aligned}
 \text{Then } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-118) \pm \sqrt{(-118)^2 - 4(13)(240)}}{2(13)} \\
 &= \frac{118 \pm \sqrt{1444}}{26} = \frac{118 \pm 38}{26} \\
 &= \frac{118+38}{26} \text{ and } \frac{118-38}{26} = 6 \text{ and } \frac{40}{13}
 \end{aligned}$$

$$\begin{aligned}
 \text{(xiv) } \frac{x-1}{x-2} + \frac{x-3}{x-4} &= 3\frac{1}{3} \\
 \Rightarrow \frac{(x-1)(x-4) + (x-2)(x-3)}{(x-2)(x-4)} &= \frac{10}{3} \\
 \Rightarrow \frac{x^2 - 4x - x + 4 + x^2 - 3x - 2x + 6}{x^2 - 4x - 2x + 8} &= \frac{10}{3}
 \end{aligned}$$

$$\Rightarrow \frac{2x^2 - 10x + 10}{x^2 - 6x + 8} = \frac{10}{3}$$

$$\Rightarrow 10x^2 - 60x + 80 = 6x^2 - 30x + 30$$

$$\Rightarrow 4x^2 - 30x + 50 = 0$$

$$\Rightarrow 2x^2 - 15x + 25 = 0$$

Here  $a=2$ ,  $b=-15$  and  $c=25$

$$\begin{aligned}
 \text{Then } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-15) \pm \sqrt{(-15)^2 - 4(2)(25)}}{2(2)} \\
 &= \frac{15 \pm \sqrt{25}}{4} = \frac{15 \pm 5}{4} \\
 &= \frac{15+5}{4} \text{ and } \frac{15-5}{4} = 5 \text{ and } \frac{5}{2}
 \end{aligned}$$

### Question 2.

Solve each of the following equations for  $x$  and give, in each case, your answer correct to one decimal place :

(i)  $x^2 - 8x + 5 = 0$

(ii)  $5x^2 + 10x - 3 = 0$

**Solution:**

(i)  $x^2 - 8x + 5 = 0$

Here  $a=1$ ,  $b=-8$  and  $c=5$ 

$$\begin{aligned} \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(5)}}{2(1)} \\ &= \frac{8 \pm \sqrt{44}}{2} = \frac{8 \pm 2\sqrt{11}}{2} = 4 \pm \sqrt{11} = 4 \pm 3.3 = 7.3 \text{ and } 0.7 \end{aligned}$$

(ii)  $5x^2 + 10x - 3 = 0$

Here  $a=5$ ,  $b=10$  and  $c=-3$ 

$$\begin{aligned} \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-10 \pm \sqrt{(10)^2 - 4(5)(-3)}}{2(5)} \\ &= \frac{-10 \pm \sqrt{160}}{10} = \frac{-10 \pm 12.6}{10} \\ &= \frac{-10 + 12.6}{10} \text{ and } \frac{-10 - 12.6}{10} = 0.26 \text{ and } -2.26 = 0.3 \text{ and } -2.3 \end{aligned}$$

**Question 3(i).**Solve each of the following equations for  $x$  and give, in each case, your answer correct to two decimal places :

(i)  $2x^2 - 10x + 5 = 0$

**Solution:**

$2x^2 - 10x + 5 = 0$

Here  $a=2$ ,  $b=-10$  and  $c=5$ 

$$\begin{aligned} \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(5)}}{2(2)} \\ &= \frac{10 \pm \sqrt{60}}{4} = \frac{10 \pm 7.75}{4} \\ &= \frac{10 + 7.75}{4} \text{ and } \frac{10 - 7.75}{4} = 4.44 \text{ and } 0.56 \end{aligned}$$

**Question 3(ii).**

Solve each of the following equations for x and give, in each case, your answer correct to two decimal places :

$$4x + \frac{6}{x} + 13 = 0$$

**Solution:**

$$4x + \frac{6}{x} + 13 = 0$$

$$\Rightarrow 4x^2 + 6 + 13x = 0$$

$$\Rightarrow 4x^2 + 13x + 6 = 0$$

Here  $a=4$ ,  $b=13$  and  $c=6$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(13) \pm \sqrt{(13)^2 - 4(4)(6)}}{2(4)}$$

$$= \frac{-13 \pm \sqrt{73}}{8} = \frac{-13 \pm 8.54}{8}$$

$$= \frac{-13 + 8.54}{8} \text{ and } \frac{-13 - 8.54}{8} = -0.56 \text{ and } -2.69$$

**Question 3(iii).**

Solve each of the following equations for x and give, in each case, your answer correct to two decimal places :

$$x^2 - 3x - 9 = 0$$

**Solution:**

$$x^2 - 3x - 9 = 0$$

Here  $a=1$ ,  $b=-3$  and  $c=-9$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{45}}{2} = \frac{3 \pm 6.70}{2}$$

$$= \frac{3 + 6.70}{2} \text{ and } \frac{3 - 6.70}{2} = 4.85 \text{ and } -1.85$$

**Question 3(iv).**

Solve each of the following equations for x and give, in each case, your answer correct to two decimal places :

$$x^2 - 5x - 10 = 0$$

**Solution:**

$$x^2 - 5x - 10 = 0$$

Here,  $a = 1$ ,  $b = -5$  and  $c = -10$

$$\begin{aligned} \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times (-10)}}{2 \times 1} \\ &= \frac{5 \pm \sqrt{25 + 40}}{2} \\ &= \frac{5 \pm \sqrt{65}}{2} \\ &= \frac{5 \pm 8.06}{2} \\ \therefore x &= \frac{5 + 8.06}{2} \text{ or } x = \frac{5 - 8.06}{2} \\ \Rightarrow x &= \frac{13.06}{2} \text{ or } x = -\frac{3.06}{2} \\ \Rightarrow x &= 6.53 \text{ or } x = -1.53 \end{aligned}$$

**Question 4.**

Solve each of the following equations for x and give, in each case, your answer correct to 3 decimal places :

(i)  $3x^2 - 12x - 1 = 0$

(ii)  $x^2 - 16x + 6 = 0$

(iii)  $2x^2 + 11x + 4 = 0$

**Solution:**

(i)  $3x^2 - 12x - 1 = 0$

Here  $a=3$ ,  $b=-12$  and  $c=-1$ 

$$\begin{aligned} \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(-1)}}{2(3)} \\ &= \frac{12 \pm \sqrt{156}}{6} = \frac{12 \pm 12.489}{6} \\ &= \frac{12 + 12.489}{6} \text{ and } \frac{12 - 12.489}{6} = 4.082 \text{ and } -0.082 \end{aligned}$$

(ii)  $x^2 - 16x + 6 = 0$

Here  $a=1$ ,  $b=-16$  and  $c=6$ 

$$\begin{aligned} \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(6)}}{2(1)} \\ &= \frac{16 \pm \sqrt{232}}{2} = \frac{16 \pm 15.231}{2} \\ &= \frac{16 + 15.231}{2} \text{ and } \frac{16 - 15.231}{2} = 15.616 \text{ and } 0.384 \end{aligned}$$

(iii)  $2x^2 + 11x + 4 = 0$

Here  $a=2$ ,  $b=11$  and  $c=4$ 

$$\begin{aligned} \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(11) \pm \sqrt{(11)^2 - 4(2)(4)}}{2(2)} \\ &= \frac{-11 \pm \sqrt{89}}{4} = \frac{-11 \pm 9.433}{4} \\ &= \frac{-11 + 9.433}{4} \text{ and } \frac{-11 - 9.433}{4} = -0.392 \text{ and } -5.108 \end{aligned}$$

**Question 5.**

Solve:

(i)  $x^4 - 2x^2 - 3 = 0$

(ii)  $x^4 - 10x^2 + 9 = 0$

**Solution:**

(i)  $x^4 - 2x^2 - 3 = 0$

$\Rightarrow x^4 - 3x^2 + x^2 - 3 = 0$

$\Rightarrow x^2(x^2 - 3) + 1(x^2 - 3) = 0$

$\Rightarrow (x^2 - 3)(x^2 + 1) = 0$

If  $x^2 - 3 = 0$  or  $x^2 + 1 = 0$

$\Rightarrow x^2 = 3$  or  $x^2 = -1$  (reject)

$\Rightarrow x = \pm\sqrt{3}$

(ii)  $x^4 - 10x^2 + 9 = 0$

$\Rightarrow x^4 - 9x^2 - x^2 + 9 = 0$

$\Rightarrow x^2(x^2 - 9) - 1(x^2 - 9) = 0$

$\Rightarrow (x^2 - 9)(x^2 - 1) = 0$

If  $x^2 - 9 = 0$  or  $x^2 - 1 = 0$

$\Rightarrow x^2 = 9$  or  $x^2 = 1$

$\Rightarrow x = \pm 3$  or  $x = \pm 1$

**Question 6.**

Solve :

(i)  $(x^2 - x)^2 + 5(x^2 - x) + 4 = 0$

(ii)  $(x^2 - 3x)^2 - 16(x^2 - 3x) - 36 = 0$

**Solution:**

$$(i) (x^2 - x)^2 + 5(x^2 - x) + 4 = 0$$

$$\text{Let } x^2 - x = y$$

$$\text{Then } y^2 + 5y + 4 = 0$$

$$\Rightarrow y^2 + 4y + y + 4 = 0$$

$$\Rightarrow y(y + 4) + 1(y + 4) = 0$$

$$\Rightarrow (y + 4)(y + 1) = 0$$

$$\text{If } y + 4 = 0 \text{ or } y + 1 = 0$$

$$\Rightarrow x^2 - x + 4 = 0 \text{ or } x^2 - x + 1 = 0$$

$$\Rightarrow x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(4)}}{2(1)} \text{ or } x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$\Rightarrow x = \frac{1 \pm \sqrt{-15}}{2} (\text{reject}) \text{ or } x = \frac{1 \pm \sqrt{-3}}{2} (\text{reject})$$

$\therefore$  Given equation has no real solution.

$$(ii) (x^2 - 3x)^2 - 16(x^2 - 3x) - 36 = 0$$

$$\text{Let } x^2 - 3x = y$$

$$\text{Then } y^2 - 16y - 36 = 0$$

$$\Rightarrow y^2 - 18y + 2y - 36 = 0$$

$$\Rightarrow y(y - 18) + 2(y - 18) = 0$$

$$\Rightarrow (y - 18)(y + 2) = 0$$

$$\text{If } y - 18 = 0 \text{ or } y + 2 = 0$$

$$\Rightarrow x^2 - 3x - 18 = 0 \text{ or } x^2 - 3x + 2 = 0$$

$$\Rightarrow x^2 - 6x + 3x - 18 = 0 \text{ or } x^2 - 2x - x + 2 = 0$$

$$\Rightarrow x(x - 6) + 3(x - 6) = 0 \text{ or } x(x - 2) - 1(x - 2) = 0$$

$$\Rightarrow (x - 6)(x + 3) = 0 \text{ or } (x - 2)(x - 1) = 0$$

$$\text{If } x - 6 = 0 \text{ or } x + 3 = 0 \text{ or } x - 2 = 0 \text{ or } x - 1 = 0$$

$$\text{then } x = 6 \text{ or } x = -3 \text{ or } x = 2 \text{ or } x = 1$$

**Question 7.**

Solve:

$$(i) \sqrt{\frac{x}{x-3}} + \sqrt{\frac{x-3}{x}} = \frac{5}{2}$$

$$(ii) \left(\frac{2x-3}{x-1}\right) - 4\left(\frac{x-1}{2x-3}\right) = 3$$

$$(iii) \left(\frac{3x+1}{x+1}\right) + \left(\frac{x+1}{3x+1}\right) = \frac{5}{2}$$

**Solution:**

$$(i) \sqrt{\frac{x}{x-3}} + \sqrt{\frac{x-3}{x}} = \frac{5}{2}$$

$$\text{Let } \sqrt{\frac{x}{x-3}} = y$$

$$\text{Then } y + \frac{1}{y} = \frac{5}{2}$$

$$\Rightarrow \frac{y^2 + 1}{y} = \frac{5}{2}$$

$$\Rightarrow 2y^2 + 2 = 5y$$

$$\Rightarrow 2y^2 - 5y + 2 = 0$$

$$\Rightarrow 2y^2 - 4y - y + 2 = 0$$

$$\Rightarrow 2y(y-2) - 1(y-2) = 0$$

$$\Rightarrow (y-2)(2y-1) = 0$$

$$\text{If } y-2=0 \quad \text{or} \quad 2y-1=0$$

$$\text{then } y=2 \quad \text{or} \quad y=\frac{1}{2}$$

$$\Rightarrow \sqrt{\frac{x}{x-3}} = 2 \quad \text{or} \quad \sqrt{\frac{x}{x-3}} = \frac{1}{2}$$

$$\Rightarrow \frac{x}{x-3} = 4 \quad \text{or} \quad \frac{x}{x-3} = \frac{1}{4}$$

$$\Rightarrow x = 4 \quad \text{or} \quad x = -1$$

$$(ii) \left( \frac{2x-3}{x-1} \right) - 4 \left( \frac{x-1}{2x-3} \right) = 3$$

$$\text{Let } \frac{2x-3}{x-1} = y$$

$$\text{then } y - \frac{4}{y} = 3$$

$$\Rightarrow \frac{y^2 - 4}{y} = 3$$

$$\Rightarrow y^2 - 4 = 3y$$

$$\Rightarrow y^2 - 3y - 4 = 0$$

$$\Rightarrow y^2 - 4y + y - 4 = 0$$

$$\Rightarrow y(y-4) + 1(y-4) = 0$$

$$\Rightarrow (y-4)(y+1) = 0$$

$$\text{If } y-4=0 \quad \text{or} \quad y+1=0$$

$$\text{then } y=4 \quad \text{or} \quad y=-1$$

$$\Rightarrow \frac{2x-3}{x-1} = 4 \quad \text{or} \quad \frac{2x-3}{x-1} = -1$$

$$\Rightarrow 4x-4 = 2x-3 \quad \text{or} \quad 2x-3 = -x+1$$

$$\Rightarrow 2x = 1 \quad \text{or} \quad 3x = 4$$

$$\Rightarrow x = \frac{1}{2} \quad \text{or} \quad x = \frac{4}{3} = 1\frac{1}{3}$$

$$(iii) \left( \frac{3x+1}{x+1} \right) + \left( \frac{x+1}{3x+1} \right) = \frac{5}{2}$$

$$\text{Let } \frac{3x+1}{x+1} = y$$

$$\text{then } y + \frac{1}{y} = \frac{5}{2}$$

$$\Rightarrow \frac{y^2 + 1}{y} = \frac{5}{2}$$

$$\Rightarrow 2y^2 + 2 = 5y$$

$$\Rightarrow 2y^2 - 5y + 2 = 0$$

$$\Rightarrow 2y^2 - 4y - y + 2 = 0$$

$$\Rightarrow 2y(y - 2) - 1(y - 2) = 0$$

$$\Rightarrow (y - 2)(2y - 1) = 0$$

$$\text{If } y - 2 = 0 \quad \text{or} \quad 2y - 1 = 0$$

$$\text{then } y = 2 \quad \text{or} \quad y = \frac{1}{2}$$

$$\Rightarrow \frac{3x + 1}{x + 1} = 2 \quad \text{or} \quad \frac{3x + 1}{x + 1} = \frac{1}{2}$$

$$\Rightarrow 3x + 1 = 2x + 2 \quad \text{or} \quad 6x + 2 = x + 1$$

$$\Rightarrow x = 1 \quad \text{or} \quad 5x = -1$$

$$\Rightarrow x = 1 \quad \text{or} \quad x = -\frac{1}{5}$$

**Question 8.**

Solve the equation  $2x - \frac{1}{x} = 7$ . Write your answer correct to two decimal places.

**Solution:**

$$2x - \frac{1}{x} = 7$$

$$\Rightarrow \frac{2x^2 - 1}{x} = 7$$

$$\Rightarrow 2x^2 - 1 = 7x$$

$$\Rightarrow 2x^2 - 7x - 1 = 0$$

Here  $a = 2$ ,  $b = -7$  and  $c = -1$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{7 \pm \sqrt{57}}{4} = \frac{7 \pm 7.55}{4}$$

$$= \frac{7 + 7.55}{4} \text{ and } \frac{7 - 7.55}{4} = 3.64 \text{ and } -0.14$$

**Question 9.**

Solve the following equation and give your answer correct to 3 significant figures:

$$5x^2 - 3x - 4 = 0$$

**Solution:**

Consider the given equation:

$$5x^2 - 3x - 4 = 0$$

Using quadratic formula, we have,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 5 \times (-4)}}{2 \times 5}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 + 80}}{2 \times 5}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{89}}{10}$$

$$\Rightarrow x = \frac{3 \pm 9.434}{10}$$

$$\Rightarrow x = 1.243 \text{ or } x = -0.643$$

**Question 10.**

Solve for x using the quadratic formula. Write your answer correct to two significant figures.

$$(x - 1)^2 - 3x + 4 = 0$$

**Solution:**

$$(x - 1)^2 - 3x + 4 = 0$$

$$\Rightarrow x^2 - 2x + 1 - 3x + 4 = 0$$

$$\Rightarrow x^2 - 5x + 5 = 0$$

Here,  $a = 1$ ,  $b = -5$  and  $c = 5$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned}
&= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 5}}{2 \times 1} \\
&= \frac{5 \pm \sqrt{25 - 20}}{2} \\
&= \frac{5 \pm \sqrt{5}}{2} \\
&= \frac{5 \pm 2.24}{2} \\
\therefore x &= \frac{5 + 2.24}{2} \text{ or } x = \frac{5 - 2.24}{2} \\
\Rightarrow x &= \frac{7.24}{2} \text{ or } x = \frac{2.76}{2} \\
\Rightarrow x &= 3.6 \text{ or } x = 1.4
\end{aligned}$$

**Question 11.**

Solve the quadratic equation  $x^2 - 3(x+3) = 0$ ; Give your answer correct to two significant figures.

**Solution:**

$$x^2 - 3(x + 3) = 0$$

$$\Rightarrow x^2 - 3x - 9 = 0$$

Comparing with  $ax^2 + bx + c$ , we get

$$a = 1, b = -3, c = -9$$

$$\text{Now, } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 + 36}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{45}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 \times 5}}{2}$$

$$\Rightarrow x = \frac{3 \pm 3\sqrt{5}}{2}$$

$$\begin{aligned} \Rightarrow x &= \frac{3+3\sqrt{5}}{2} \text{ or } x = \frac{3-3\sqrt{5}}{2} \\ \Rightarrow x &= \frac{3+3 \times 2.236}{2} \text{ or } x = \frac{3-3 \times 2.236}{2} \\ \Rightarrow x &= \frac{3+6.708}{2} \text{ or } x = \frac{3-6.708}{2} \\ \Rightarrow x &= \frac{9.708}{2} \text{ or } x = \frac{-3.708}{2} \\ \Rightarrow x &= 4.854 \text{ or } x = -1.854 \\ \Rightarrow x &= 4.9 \text{ or } x = -1.9 \end{aligned}$$

## Exercise 5E

### Question 1.

Solve:

$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0; \quad x \neq 3, \quad x \neq -\frac{3}{2}$$

**Solution:**

$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0; \quad x \neq 3, \quad x \neq -\frac{3}{2}$$

$$\Rightarrow \frac{2x(2x+3) + 1(x-3) + 3x+9}{(x-3)(2x+3)} = 0$$

$$\Rightarrow 4x^2 + 6x + x - 3 + 3x + 9 = 0$$

$$\Rightarrow 4x^2 + 10x + 6 = 0$$

$$\Rightarrow 4x^2 + 4x + 6x + 6 = 0$$

$$\Rightarrow 4x(x+1) + 6(x+1) = 0$$

$$\Rightarrow (x+1)(4x+6) = 0$$

$$\Rightarrow x+1=0 \quad \text{or} \quad 4x+6=0$$

$$\Rightarrow x = -1 \quad \text{or} \quad x = \frac{-6}{4} = \frac{-3}{2} \text{ (reject)}$$

### Question 2.

Solve:  $(2x+3)^2=81$

**Solution:**

$$(2x + 3)^2 = 81$$

$$\Rightarrow 2x + 3 = \pm 9$$

$$\Rightarrow 2x + 3 = 9 \quad \text{and} \quad 2x + 3 = -9$$

$$\Rightarrow 2x = 6 \quad \text{and} \quad 2x = -12$$

$$\Rightarrow x = 3 \quad \text{and} \quad x = -6$$

**Question 3.**Solve:  $a^2x^2 - b^2 = 0$ **Solution:**

$$a^2x^2 - b^2 = 0$$

$$\Rightarrow (ax)^2 - b^2 = 0$$

$$\Rightarrow (ax + b)(ax - b) = 0$$

$$\text{If } ax + b = 0 \quad \text{and} \quad ax - b = 0$$

$$\text{then } x = \frac{-b}{a} \quad \text{and} \quad x = \frac{b}{a}$$

**Question 4.**

$$\text{Solve: } x^2 - \frac{11}{4}x + \frac{15}{8} = 0$$

**Solution:**

$$x^2 - \frac{11}{4}x + \frac{15}{8} = 0$$

$$\Rightarrow \frac{8x^2 - 22x + 15}{8} = 0$$

$$\Rightarrow 8x^2 - 22x + 15 = 0$$

$$\Rightarrow 8x^2 - 12x - 10x + 15 = 0$$

$$\Rightarrow 4x(2x - 3) - 5(2x - 3) = 0$$

$$\Rightarrow (2x - 3)(4x - 5) = 0$$

$$\Rightarrow 2x - 3 = 0 \quad \text{or} \quad 4x - 5 = 0$$

$$\Rightarrow x = \frac{3}{2} \quad \text{or} \quad x = \frac{5}{4}$$

**Question 5.**

$$x + \frac{4}{x} = -4; x \neq 0$$

**Solution:**

$$x + \frac{4}{x} = -4$$

$$\Rightarrow \frac{x^2 + 4}{x} = -4$$

$$\Rightarrow x^2 + 4 = -4x$$

$$\Rightarrow x^2 + 4x + 4 = 0$$

$$\Rightarrow (x + 2)^2 = 0$$

$$\Rightarrow x + 2 = 0$$

$$\Rightarrow x = -2$$

**Question 6.**

Solve:  $2x^4 - 5x^2 + 3 = 0$

**Solution:**

$$2x^4 - 5x^2 + 3 = 0$$

$$\Rightarrow 2x^4 - 3x^2 - 2x^2 + 3 = 0$$

$$\Rightarrow x^2(2x^2 - 3) - 1(2x^2 - 3) = 0$$

$$\Rightarrow (2x^2 - 3)(x^2 - 1) = 0$$

If  $2x^2 - 3 = 0$  or  $x^2 - 1 = 0$

then  $x^2 = \frac{3}{2}$  or  $x^2 = 1$

$$\Rightarrow x = \pm \sqrt{\frac{3}{2}} \text{ or } x = \pm 1$$

**Question 7.**Solve:  $x^4 - 2x^2 - 3 = 0$ .**Solution:**

$$x^4 - 2x^2 - 3 = 0$$

$$\Rightarrow x^4 - 3x^2 + x^2 - 3 = 0$$

$$\Rightarrow x^2(x^2 - 3) + 1(x^2 - 3) = 0$$

$$\Rightarrow (x^2 - 3)(x^2 + 1) = 0$$

$$\text{If } x^2 - 3 = 0 \quad \text{or} \quad x^2 + 1 = 0$$

$$\text{then } x^2 = 3 \quad \text{or} \quad x^2 = -1(\text{reject})$$

$$\Rightarrow x = \pm\sqrt{3}$$

**Question 8.**

$$\text{Solve : } 9\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) - 52 = 0$$

**Solution:**

$$9\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) - 52 = 0$$

$$\text{Let } x + \frac{1}{x} = y$$

squaring on both sides

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$$

Putting these values in the given equation

$$9(y^2 - 2) - 9y - 52 = 0$$

$$\Rightarrow 9y^2 - 18 - 9y - 52 = 0$$

$$\Rightarrow 9y^2 - 9y - 70 = 0$$

$$\Rightarrow 9y^2 - 30y + 21y - 70 = 0$$

$$\Rightarrow 3y(3y - 10) + 7(3y - 10) = 0$$

$$\Rightarrow (3y - 10)(3y + 7) = 0$$

$$\Rightarrow 3y - 10 = 0 \quad \text{or} \quad 3y + 7 = 0$$

$$\Rightarrow y = \frac{10}{3} \quad \text{or} \quad y = \frac{-7}{3}$$

$$\Rightarrow x + \frac{1}{x} = \frac{10}{3} \quad \text{or} \quad x + \frac{1}{x} = \frac{-7}{3}$$

$$\Rightarrow \frac{x^2 + 1}{x} = \frac{10}{3} \quad \text{or} \quad \frac{x^2 + 1}{x} = \frac{-7}{3}$$

$$\Rightarrow 3x^2 - 10x + 3 = 0 \quad \text{or} \quad 3x^2 + 7x + 3 = 0$$

$$\Rightarrow 3x^2 - 9x - x + 3 = 0 \quad \text{or} \quad x = \frac{-7 \pm \sqrt{(-7)^2 - 4(3)(3)}}{2(3)}$$

$$\Rightarrow 3x(x - 3) - 1(x - 3) = 0 \quad \text{or} \quad x = \frac{-7 \pm \sqrt{13}}{6}$$

$$\Rightarrow (x - 3)(3x - 1) = 0$$

$$\Rightarrow x = 3 \quad \text{and} \quad x = \frac{1}{3}$$

$$\Rightarrow (3y - 10)(3y + 7) = 0$$

$$\Rightarrow 3y - 10 = 0 \quad \text{or} \quad 3y + 7 = 0$$

$$\Rightarrow y = \frac{10}{3} \quad \text{or} \quad y = \frac{-7}{3}$$

$$\Rightarrow x + \frac{1}{x} = \frac{10}{3} \quad \text{or} \quad x + \frac{1}{x} = \frac{-7}{3}$$

$$\Rightarrow \frac{x^2 + 1}{x} = \frac{10}{3} \quad \text{or} \quad \frac{x^2 + 1}{x} = \frac{-7}{3}$$

$$\Rightarrow 3x^2 - 10x + 3 = 0 \quad \text{or} \quad 3x^2 + 7x + 3 = 0$$

$$\Rightarrow 3x^2 - 9x - x + 3 = 0 \quad \text{or} \quad x = \frac{-7 \pm \sqrt{(-7)^2 - 4(3)(3)}}{2(3)}$$

$$\Rightarrow 3x(x - 3) - 1(x - 3) = 0 \quad \text{or} \quad x = \frac{-7 \pm \sqrt{13}}{6}$$

$$\Rightarrow (x - 3)(3x - 1) = 0$$

$$\Rightarrow x = 3 \quad \text{and} \quad x = \frac{1}{3}$$

**Question 9.**

$$\text{Solve : } 2\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right) = 11$$

**Solution:**

$$2\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right) = 11$$

$$\text{Let } x + \frac{1}{x} = y$$

squaring on both sides

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$$

Putting these values in the given equation

$$2(y^2 - 2) - y = 11$$

$$\Rightarrow 2y^2 - 4 - y - 11 = 0$$

$$\Rightarrow 2y^2 - y - 15 = 0$$

$$\Rightarrow 2y^2 - 6y + 5y - 15 = 0$$

$$\Rightarrow 2y(y - 3) + 5(y - 3) = 0$$

$$\Rightarrow (y - 3)(2y + 5) = 0$$

$$\text{If } y - 3 = 0 \quad \text{or} \quad 2y + 5 = 0$$

$$\text{then } y = 3 \quad \text{or} \quad y = -\frac{5}{2}$$

$$\Rightarrow x + \frac{1}{x} = 3$$

$$\Rightarrow \frac{x^2 + 1}{x} = 3$$

$$\Rightarrow x^2 - 3x + 1 = 0$$

$$\Rightarrow x = \frac{-3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$\text{or } x + \frac{1}{x} = -\frac{5}{2}$$

$$\text{or } \frac{x^2 + 1}{x} = -\frac{5}{2}$$

$$\text{or } 2x^2 + 5x + 2 = 0$$

$$\text{or } 2x^2 + 4x + x + 2 = 0$$

$$\Rightarrow x = \frac{-3 \pm \sqrt{5}}{2}$$

$$\text{or } 2x(x+2)+1(x+2)=0$$

$$\text{or } (x+2)(2x+1)=0$$

$$\text{then } x = -2 \text{ and } x = \frac{-1}{2}$$

**Question 10.**

$$\text{Solve : } \left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0$$

**Solution:**

$$\left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0$$

$$\text{Let } x - \frac{1}{x} = y$$

squaring on both sides

$$x^2 + \frac{1}{x^2} - 2 = y^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = y^2 + 2$$

Putting these values in the given equation

$$(y^2 + 2) - 3y - 2 = 0$$

$$\Rightarrow y^2 - 3y = 0$$

$$\Rightarrow y(y - 3) = 0$$

$$\text{If } y = 0$$

$$\text{then } y = 0$$

$$\Rightarrow x - \frac{1}{x} = 0$$

$$\Rightarrow \frac{x^2 - 1}{x} = 0$$

$$\Rightarrow x^2 - 1 = 0$$

$$\Rightarrow (x + 1)(x - 1) = 0$$

$$\Rightarrow x = -1 \text{ and } x = 1$$

$$\text{or } y - 3 = 0$$

$$\text{or } y = 3$$

$$\text{or } x - \frac{1}{x} = 3$$

$$\text{or } \frac{x^2 - 1}{x} = 3$$

$$\text{or } x^2 - 3x - 1 = 0$$

$$\text{or } x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$$

$$\text{or } x = \frac{3 \pm \sqrt{13}}{2}$$

**Question 11.**Solve :  $(x^2 + 5x + 4)(x^2 + 5x + 6) = 120$ **Solution:**

$$(x^2 + 5x + 4)(x^2 + 5x + 6) = 120$$

$$\text{Let } x^2 + 5x = y$$

$$\text{then } (y+4)(y+6) = 120$$

$$\Rightarrow y^2 + 6y + 4y + 24 - 120 = 0$$

$$\Rightarrow y^2 + 10y - 96 = 0$$

$$\Rightarrow y^2 + 16y - 6y - 96 = 0$$

$$\Rightarrow y(y + 16) - 6(y + 16) = 0$$

$$\Rightarrow (y + 16)(y - 6) = 0$$

$$\text{then } y = -16 \quad \text{or} \quad y = 6$$

$$\Rightarrow x^2 + 5x + 16 = 0 \quad \text{or} \quad x^2 + 5x - 6 = 0$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(16)}}{2(1)} \quad \text{or} \quad x^2 + 6x - x - 6 = 0$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{-39}}{2} \quad \text{or } x(x+6) - 1(x+6) = 0$$

(reject)

$$\text{or } (x+6)(x-1) = 0$$

$$\text{then } x = -6 \quad \text{and} \quad x = 1$$

**Question 12.**

Solve each of the following equations, giving answer upto two decimal places.

(i)  $x^2 - 5x - 10 = 0$  (ii)  $3x^2 - x - 7 = 0$ **Solution:**

$$(i)x^2 - 5x - 10 = 0$$

Here  $a = 1$ ,  $b = -5$  and  $c = -10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-10)}}{2(1)}$$

$$\Rightarrow x = \frac{5 \pm \sqrt{65}}{2} = \frac{5 \pm 8.06}{2}$$

$$\Rightarrow x = \frac{13.06}{2} \text{ and } \frac{-3.06}{2} = 6.53 \text{ and } -1.53$$

$$(ii)3x^2 - x - 7 = 0$$

Here  $a = 3$ ,  $b = -1$  and  $c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-7)}}{2(3)}$$

$$\Rightarrow x = \frac{1 \pm \sqrt{85}}{6} = \frac{1 \pm 9.22}{6}$$

$$\Rightarrow x = \frac{10.22}{6} \text{ and } \frac{-8.22}{6} = 1.70 \text{ and } -1.37$$

### Question 13.

$$\text{Solve : } \left(\frac{x}{x+2}\right)^2 - 7\left(\frac{x}{x+2}\right) + 12 = 0; x \neq -2$$

**Solution:**

$$\left(\frac{x}{x+2}\right)^2 - 7\left(\frac{x}{x+2}\right) + 12 = 0; x \neq -2$$

$$\text{Let } \frac{x}{x+2} = y$$

$$\text{then } y^2 - 7y + 12 = 0$$

$$\Rightarrow y^2 - 4y - 3y + 12 = 0$$

$$\Rightarrow y(y - 4) - 3(y - 4) = 0$$

$$\Rightarrow (y - 4)(y - 3) = 0$$

$$\text{then } y=4 \text{ and } y=3$$

$$\Rightarrow \frac{x}{x+2} = 4 \text{ and } \frac{x}{x+2} = 3$$

$$\Rightarrow 4x + 8 = x \text{ and } 3x + 6 = x$$

$$\Rightarrow x = \frac{-8}{3} \text{ and } x = -3$$

**Question 14.**

Solve :

(i)  $x^2 - 11x - 12 = 0$ ; when  $x \in \mathbb{N}$

(ii)  $x^2 - 4x - 12 = 0$ ; when  $x \in \mathbb{I}$

(iii)  $2x^2 - 9x + 10 = 0$ ; when  $x \in \mathbb{Q}$

**Solution:**

(i)  $x^2 - 11x - 12 = 0$

$$\Rightarrow x^2 - 12x + x - 12 = 0$$

$$\Rightarrow x(x - 12) + 1(x - 12) = 0$$

$$\Rightarrow (x - 12)(x + 1) = 0$$

then  $x = 12$  and  $x = -1$

Since  $x \in \mathbb{N}$ , then  $x = 12$

(ii)  $x^2 - 4x - 12 = 0$

$$\Rightarrow x^2 - 6x + 2x - 12 = 0$$

$$\Rightarrow x(x - 6) + 2(x - 6) = 0$$

$$\Rightarrow (x - 6)(x + 2) = 0$$

then  $x = 6$  and  $x = -2$

Since  $x \in \mathbb{I}$ , then  $x = 6$  and  $-2$

(iii)  $2x^2 - 9x + 10 = 0$

$$\Rightarrow 2x^2 - 5x - 4x - 10 = 0$$

$$\Rightarrow x(2x - 5) - 2(2x - 5) = 0$$

$$\Rightarrow (2x - 5)(x - 2) = 0$$

then  $x = \frac{5}{2}$  and  $x = 2$

Since  $x \in \mathbb{Q}$ , then  $x = \frac{5}{2}$  and  $2$

**Question 15.**Solve :  $(a + b)^2x^2 - (a + b)x - 6 = 0$ ;  $a + b \neq 0$ **Solution:**

$$\begin{aligned}
(a + b)^2x^2 - (a + b)x - 6 &= 0; \quad a + b \neq 0 \\
\Rightarrow (a + b)^2x^2 - 3(a + b)x + 2(a + b)x - 6 &= 0 \\
\Rightarrow (a + b)x[(a + b)x - 3] + 2[(a + b)x - 3] &= 0 \\
\Rightarrow [(a + b)x - 3][(a + b)x + 2] &= 0 \\
\Rightarrow (a + b)x - 3 = 0 \quad \text{or} \quad (a + b)x + 2 = 0 \\
\Rightarrow x = \frac{3}{a + b} \quad \text{or} \quad x = \frac{-2}{a + b}
\end{aligned}$$

**Question 16.**Solve :  $\frac{1}{p} + \frac{1}{q} + \frac{1}{x} = \frac{1}{x + p + q}$ **Solution:**

$$\begin{aligned}
\frac{1}{p} + \frac{1}{q} + \frac{1}{x} &= \frac{1}{x + p + q} \\
\Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{x} - \frac{1}{x + p + q} &= 0 \\
\Rightarrow \frac{q + p}{pq} + \frac{x + p + q - x}{x(x + p + q)} &= 0 \\
\Rightarrow \frac{q + p}{pq} + \frac{p + q}{x(x + p + q)} &= 0 \\
\Rightarrow (p + q) \left[ \frac{1}{pq} + \frac{1}{x^2 + px + qx} \right] &= 0 \\
\Rightarrow (p + q) \left[ \frac{x^2 + px + qx + pq}{pq(x^2 + px + qx)} \right] &= 0 \\
\Rightarrow x^2 + px + qx + pq &= 0 \\
\Rightarrow x(x + p) + q(x + p) &= 0 \\
\Rightarrow (x + p)(x + q) &= 0 \\
\Rightarrow x = -p \quad \text{and} \quad x = -q
\end{aligned}$$

**Question 17.**

Solve :

(i)  $x(x+1) + (x+2)(x+3) = 42$

(ii)  $\frac{1}{x+1} - \frac{2}{x+2} = \frac{3}{x+3} - \frac{4}{x+4}$

**Solution:**

(i)  $x(x+1) + (x+2)(x+3) = 42$

$\Rightarrow x^2 + x + x^2 + 3x + 2x + 6 - 42 = 0$

$\Rightarrow 2x^2 + 6x - 36 = 0$

$\Rightarrow 2x^2 + 12x - 6x - 36 = 0$

$\Rightarrow 2x(x+6) - 6(x+6) = 0$

$\Rightarrow (x+6)(2x-6) = 0$

If  $x+6 = 0$  or  $2x-6 = 0$

then  $x = -6$  or  $x = 3$

(ii)  $\frac{1}{x+1} - \frac{2}{x+2} = \frac{3}{x+3} - \frac{4}{x+4}$

$\Rightarrow \frac{1(x+2) - 2(x+1)}{(x+1)(x+2)} = \frac{3(x+4) - 4(x+3)}{(x+3)(x+4)}$

$\Rightarrow \frac{-x}{x^2 + 3x + 2} = \frac{-x}{x^2 + 7x + 12}$

$\Rightarrow -x[x^2 + 3x + 2 = x^2 + 7x + 12]$

$\Rightarrow -x[-4x = 10]$

$\Rightarrow x = 0$  and  $x = \frac{-10}{4} = -2.5$

**Question 18.**For each equation, given below, find the value of  $m$  so that the equation has equal roots. Also, find the solution of each equation :

(i)  $(m-3)x^2 - 4x + 1 = 0$

(ii)  $3x^2 + 12x + (m+7) = 0$

(iii)  $x^2 - (m+2)x + (m+5) = 0$

**Solution:**

$$(i)(m-3)x^2 - 4x + 1 = 0$$

Here  $a=(m-3)$ ,  $b= -4$  and  $c= 1$

Given equation has equal roots  
then  $D=0$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow (-4)^2 - 4(m-3)(1) = 0$$

$$\Rightarrow 16 - 4m + 12 = 0$$

$$\Rightarrow -4m = -28$$

$$\Rightarrow m = 7$$

Put value of  $m$  in given equation

$$4x^2 - 4x + 1 = 0$$

$$\Rightarrow (2x - 1)^2 = 0$$

$$\Rightarrow 2x - 1 = 0$$

$$\Rightarrow x = \frac{1}{2}$$

$$(ii)3x^2 + 12x + (m+7) = 0$$

Here  $a=3$ ,  $b= 12$  and  $c= m+7$

Given equation has equal roots  
then  $D=0$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow (12)^2 - 4(3)(m+7) = 0$$

$$\Rightarrow 144 - 12m - 84 = 0$$

$$\Rightarrow -12m = -60$$

$$\Rightarrow m = 5$$

Put value of m in given equation

$$3x^2 + 12x + 12 = 0$$

$$\Rightarrow x^2 + 4x + 4 = 0$$

$$\Rightarrow (x + 2)^2 = 0$$

$$\Rightarrow x + 2 = 0$$

$$\Rightarrow x = -2$$

$$(iii) x^2 - (m + 2)x + (m + 5) = 0$$

Here  $a=1$ ,  $b=-(m+2)$  and  $c=m+5$

Given equation has equal roots  
then  $D=0$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow [-(m + 2)]^2 - 4(1)(m + 5) = 0$$

$$\Rightarrow m^2 + 4m + 4 - 4m - 20 = 0$$

$$\Rightarrow m^2 - 16 = 0$$

$$\Rightarrow m^2 = 16$$

$$\Rightarrow m = \pm 4$$

Put value of m in given equation

$$x^2 - 6x + 9 = 0 \quad \text{or} \quad x^2 + 2x + 1 = 0$$

$$\Rightarrow (x - 3)^2 = 0 \quad \text{or} \quad (x + 1)^2 = 0$$

$$\Rightarrow x - 3 = 0 \quad \text{or} \quad x + 1 = 0$$

$$\Rightarrow x = 3 \quad \text{or} \quad x = -1$$

### Question 19.

Without solving the following quadratic equation, find the value of p for which the roots are equal.

$$px^2 - 4x + 3 = 0$$

**Solution:**

$$px^2 - 4x + 3 = 0$$

Here  $a=p$ ,  $b=-4$  and  $c=3$

Given equation has equal roots

then  $D=0$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow [-4]^2 - 4(p)(3) = 0$$

$$\Rightarrow 16 - 12p = 0$$

$$\Rightarrow -12p = -16$$

$$\Rightarrow p = \frac{-16}{-12} = \frac{4}{3}$$

**Question 20.**

Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots.

$$x^2 + 2(m - 1)x + (m + 5) = 0$$

**Solution:**

Consider the given equation:

$$x^2 + 2(m - 1)x + (m + 5) = 0$$

The nature of the roots of a quadratic equation

$ax^2 + bx + c = 0$ , depends entirely on the

value of its discriminant  $b^2 - 4ac$ .

If  $a$ ,  $b$  and  $c$  are real numbers and  $a \neq 0$ ,

then discriminant:

(i)  $b^2 - 4ac = 0 \Rightarrow$  the roots are real and equal.

(ii)  $b^2 - 4ac > 0 \Rightarrow$  the roots are real and unequal.

(i)  $b^2 - 4ac < 0 \Rightarrow$  the roots are imaginary (not real).

Since the roots of the given equation are real and equal,

we have,

$$b^2 - 4ac = 0$$

$$\Rightarrow \{2(m - 1)\}^2 - 4 \times 1 \times (m + 5) = 0$$

$$\begin{aligned}
&\Rightarrow 4(m^2 + 1 - 2m) - 4(m + 5) = 0 \\
&\Rightarrow 4m^2 + 4 - 8m - 4m - 20 = 0 \\
&\Rightarrow 4m^2 - 12m - 16 = 0 \\
&\Rightarrow m^2 - 3m - 4 = 0 \\
&\Rightarrow m^2 - 4m + m - 4 = 0 \\
&\Rightarrow m(m - 4) + 1(m - 4) = 0 \\
&\Rightarrow (m + 1)(m - 4) = 0 \\
&\Rightarrow m + 1 = 0 \text{ or } m - 4 = 0 \\
&\Rightarrow m = -1 \text{ or } m = 4
\end{aligned}$$

## Exercise 5F

### Solution 1(i)

$$\begin{aligned}
\text{Given: } (x + 5)(x - 5) &= 24 \\
\Rightarrow x^2 - 5^2 &= 24 \quad \dots \text{ since } (a - b)(a + b) = a^2 - b^2 \\
\Rightarrow x^2 - 25 &= 24 \\
\Rightarrow x^2 &= 49 \\
\Rightarrow x &= \pm 7
\end{aligned}$$

### Solution 1(ii)

$$\begin{aligned}
\text{Given: } 3x^2 - 2\sqrt{6}x + 2 &= 0 \\
\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 &= 0 \\
\Rightarrow \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) &= 0 \\
\Rightarrow (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) &= 0 \\
\Rightarrow (\sqrt{3}x - \sqrt{2}) = 0 \text{ or } (\sqrt{3}x - \sqrt{2}) &= 0 \\
\Rightarrow x = \sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}
\end{aligned}$$

### Solution 1(iii)

$$\begin{aligned}
\text{Given: } 3\sqrt{2}x^2 - 5x - \sqrt{26} &= 0 \\
\Rightarrow 3\sqrt{2}x^2 - 6x + x - \sqrt{2} &= 0 \\
\Rightarrow 3\sqrt{2}x(x - \sqrt{2}) + (x - \sqrt{2}) &= 0 \\
\Rightarrow (3\sqrt{2}x + 1)(x - \sqrt{2}) &= 0 \\
\Rightarrow x = -\frac{1}{3\sqrt{2}} \text{ or } x = \sqrt{2}
\end{aligned}$$

**Question 2.**

One root of the quadratic equation  $8x^2 + mx + 15 = 0$  is  $\frac{3}{4}$ . Find the value of  $m$ . Also, find the other root of the equation.

**Solution:**

Given quadratic equation is  $8x^2 + mx + 15 = 0$  .... (i)

One of the roots of (i) is  $\frac{3}{4}$ , so it satisfies (i)

$$\Rightarrow 8\left(\frac{3}{4}\right)^2 + m\left(\frac{3}{4}\right) + 15 = 0$$

$$\Rightarrow \frac{9}{2} + 15 + m\left(\frac{3}{4}\right) = 0$$

$$\Rightarrow m\left(\frac{3}{4}\right) = -\frac{39}{2}$$

$$\Rightarrow m = -26$$

So, the equation (i) becomes  $8x^2 - 26x + 15 = 0$

$$\Rightarrow 8x^2 - 20x - 6x + 15 = 0$$

$$\Rightarrow 4x(2x - 5) - 3(2x - 5) = 0$$

$$\Rightarrow (4x - 3)(2x - 5) = 0$$

$$\Rightarrow x = \frac{3}{4} \text{ or } x = \frac{5}{2}$$

$$\Rightarrow x = \frac{3}{4}, \frac{5}{2}$$

Hence, the other root is  $\frac{5}{2}$

**Question 3.**

One root of the quadratic equation  $x^2 - 3x - 2ax - 6a = 0$  is  $-3$ , find its other root.

**Solution:**

Given quadratic equation is .... (i)

One of the roots of (i) is  $-3$ , so it satisfies (i)

$$\Rightarrow x^2 - 3x - 2ax - 6a = 0$$

$$\Rightarrow x(x + 3) - 2a(x + 3) = 0$$

$$\Rightarrow (x - 2a)(x + 3) = 0$$

$$\Rightarrow x = -3, 2a$$

Hence, the other root is  $2a$ .

**Question 4.**

If  $p - 15 = 0$  and  $2x^2 + 15x + 15 = 0$ ; find the values of  $x$ .

**Solution:**

Given i.e  $p - 15 = 0$  i.e.  $p = 15$

So, the given quadratic equation becomes

$$2x^2 + 15x + 15 = 0$$

$$\Rightarrow 2x + 10x + 5x + 15 = 0$$

$$\Rightarrow 2x(x + 5) + 5(x + 5)$$

$$\Rightarrow (2x + 5)(x + 5) = 0$$

$$\Rightarrow x = -5, -\frac{5}{2}$$

Hence, the values of x are -5 and  $-\frac{5}{2}$

### Question 5.

Find the solution of the equation  $2x^2 - mx - 25n = 0$ ; if  $m + 5 = 0$  and  $n - 1 = 0$ .

#### Solution:

Given quadratic equation is  $2x^2 - mx - 25n = 0$  ..... (i)

Also, given and  $m + 5 = 0$  and  $n - 1 = 0$

$$\Rightarrow m = -5 \text{ and } n = 1$$

So, the equation (i) becomes

$$2x^2 + 5x + 25 = 0$$

$$\Rightarrow 2x + 10x - 5x - 25 = 0$$

$$\Rightarrow 2x(x + 5) - 5(x + 5) = 0$$

$$\Rightarrow (x + 5)(2x - 5) = 0$$

$$\Rightarrow x = -5, \frac{5}{2}$$

Hence, the solution of given quadratic equation are x and  $\frac{5}{2}$

### Question 6.

If m and n are roots of the equation  $\frac{1}{x} - \frac{1}{x-2} = 3$  where  $x \neq 0$  and  $x \neq 2$ ; find  $m \times n$ .

#### Solution:

Given quadratic equation is  $\frac{1}{x} - \frac{1}{x-2} = 3$

$$\Rightarrow x - 2 - x = 3x(x - 2)$$

$$\Rightarrow -2 = 3x^2 - 6x$$

$$\Rightarrow 3x^2 - 6x + 2 = 0$$

$$\Rightarrow x = \frac{6 \pm \sqrt{6^2 - 4(3)(2)}}{2 \times 3}$$

$$\Rightarrow x = \frac{6 \pm \sqrt{12}}{2 \times 3}$$

$$\Rightarrow x = \frac{\sqrt{3} \pm 1}{\sqrt{3}}$$

Since, m and n are roots of the equation, we have

$$\Rightarrow m = \frac{\sqrt{3}+1}{\sqrt{3}} \quad \text{and} \quad n = \frac{\sqrt{3}-1}{\sqrt{3}}$$

$$\Rightarrow m \times n = \left( \frac{\sqrt{3}+1}{\sqrt{3}} \right) \left( \frac{\sqrt{3}-1}{\sqrt{3}} \right) = \frac{2}{3}$$

$$m \times n = \frac{2}{3}$$

### Question 7.

Solve, using formula :

$$x^2 + x - (a+2)(a+1) = 0$$

### Solution:

Given quadratic equation is  $x^2 + x - (a+2)(a+1) = 0$

Using quadratic formula,

$$\Rightarrow x = \frac{-1 \pm \sqrt{1^2 + 4(a+2)(a+1)}}{2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 4(a^2 + 3a + 2)}}{2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{4a^2 + 12a + 9}}{2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{(2a+3)^2}}{2}$$

$$\Rightarrow x = \frac{-1 \pm (2a+3)}{2}$$

$$\Rightarrow x = \frac{-1 + (2a+3)}{2} \quad \text{or} \quad x = \frac{-1 - (2a+3)}{2}$$

$$\Rightarrow x = \frac{2a+2}{2} \quad \text{or} \quad x = \frac{-2a-4}{2}$$

$$\Rightarrow x = \frac{2(a+1)}{2} \quad \text{or} \quad x = \frac{2(-a-2)}{2}$$

$$\Rightarrow x = a+1 \quad \text{or} \quad x = -a-2 = -(a+2)$$

### Question 8.

Solve the quadratic equation  $8x^2 - 14x + 3 = 0$

(i) When  $x \in \mathbb{I}$  (integers)

(ii) When  $x \in \mathbb{Q}$  (rational numbers)

**Solution:**

Given quadratic equation is  $8x^2 - 14x + 3 = 0$

$$\Rightarrow 8x^2 - 12x - 2x + 3 = 0$$

$$\Rightarrow 4x(2x - 3) - (2x - 3) = 0$$

$$\Rightarrow (4x - 1)(2x - 3) = 0$$

$$\Rightarrow x = \frac{3}{2} \text{ or } x = \frac{1}{4}$$

(i) When  $x \in I$ , the equation  $8x^2 - 14x + 3 = 0$  has no roots

(ii) When  $x \in Q$  the roots of  $8x^2 - 14x + 3 = 0$  are

$$x = \frac{3}{2} \text{ or } x = \frac{1}{4}$$

**Question 9.**

Find the value of  $m$  for which the equation  $(m + 4)^2 + (m + 1)x + 1 = 0$  has real and equal roots.

**Solution:**

Given quadratic equation is  $(m + 4)^2 + (m + 1)x + 1 = 0$

The quadratic equation has real and equal roots if its discriminant is zero.

$$\Rightarrow D = b^2 - 4ac = 0$$

$$\Rightarrow (m + 1)^2 - 4(m + 4)(1) = 0$$

$$\Rightarrow m^2 + 2m + 1 - 4m - 16 = 0$$

$$\Rightarrow m^2 - 2m - 15 = 0$$

$$\Rightarrow m^2 - 5m + 3m - 15 = 0$$

$$\Rightarrow m(m - 5) + 3(m - 5) = 0$$

$$\Rightarrow (m - 5)(m + 3) = 0$$

$$\Rightarrow m = 5 \text{ or } m = -3$$

**Question 10.**

Find the values of  $m$  for which equation  $3x^2 + mx + 2 = 0$  has equal roots. Also, find the roots of the given equation.

**Solution:**

Given quadratic equation is  $3x^2 + mx + 2 = 0$  .... (i)

The quadratic equation has equal roots if its discriminant is zero

$$\Rightarrow D = b^2 - 4ac = 0$$

$$\Rightarrow m^2 - 4(2)(3) = 0$$

$$\Rightarrow m^2 = 24$$

$$\Rightarrow m = \pm 2\sqrt{6}$$

When  $m = 2\sqrt{6}$ , equation (i) becomes

$$\begin{aligned}
 3x^2 + 2\sqrt{6}x + 2 &= 0 \\
 \Rightarrow (\sqrt{3}x + \sqrt{2})^2 &= 0 \\
 \Rightarrow x &= -\frac{\sqrt{2}}{\sqrt{3}} = -\frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{6}}{3}
 \end{aligned}$$

When  $m = -2\sqrt{6}$ , equation (i) becomes

$$\begin{aligned}
 3x^2 - 2\sqrt{6}x + 2 &= 0 \\
 \Rightarrow (\sqrt{3}x - \sqrt{2})^2 &= 0 \\
 \Rightarrow x &= \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3} \\
 \therefore x &= -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}
 \end{aligned}$$

### Question 11.

Find the value of  $k$  for which equation  $4x^2 + 8x - k = 0$  has real roots.

#### Solution:

Given quadratic equation is  $4x^2 + 8x - k = 0$  .... (i)

The quadratic equation has real roots if its discriminant is greater than or equal to zero

$$\Rightarrow D = b^2 - 4ac \geq 0$$

$$\Rightarrow 8^2 - 4(4)(-k) \geq 0$$

$$\Rightarrow 64 + 16k \geq 0$$

$$\Rightarrow 16k \geq -64$$

$$\Rightarrow k \geq -4$$

Hence, the given quadratic equation has real roots for  $k \geq -4$

### Question 12.

Find, using quadratic formula, the roots of the following quadratic equations, if they exist

(i)  $3x^2 - 5x + 2 = 0$

(ii)  $x^2 + 4x + 5 = 0$

#### Solution:

(i) Given quadratic equation is  $3x^2 - 5x + 2 = 0$

$$D = b^2 - 4ac = (-5)^2 - 4(3)(2) = 25 - 24 = 1$$

Since  $D > 0$ , the roots of the given quadratic equation are real and distinct.

Using quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(2)}}{2(3)}$$

$$\Rightarrow x = \frac{5 \pm \sqrt{25 - 24}}{6}$$

$$\Rightarrow x = \frac{5 \pm 1}{6}$$

$$\Rightarrow x = \frac{5+1}{6} \text{ or } x = \frac{5-1}{6}$$

$$\Rightarrow x = \frac{6}{6} \text{ or } x = \frac{4}{6}$$

$$\Rightarrow x = 1 \text{ or } x = \frac{2}{3}$$

(ii) Given quadratic equation is  $x^2 + 4x + 5 = 0$

$$D = b^2 - 4ac = (4)^2 - 4(1)(5) = 16 - 20 = -4$$

Since  $D < 0$ , the roots of the given quadratic equation does not exist.

**Solution 13:**

(i) Given quadratic equation is  $\frac{1}{18-x} - \frac{1}{18+x} = \frac{1}{24}$

$$\Rightarrow \frac{(18+x) - (18-x)}{(18+x)(18-x)} = \frac{1}{24}$$

$$\Rightarrow \frac{2x}{18^2 - x^2} = \frac{1}{24}$$

$$\Rightarrow 48x = 324 - x^2$$

$$\Rightarrow x^2 + 48x - 324 = 0$$

$$\Rightarrow x^2 + 54x - 6x - 324 = 0$$

$$\Rightarrow x(x + 54) - 6(x + 54) = 0$$

$$\Rightarrow (x + 54)(x - 6) = 0$$

$$\Rightarrow x = -54 \text{ or } x = 6$$

But as  $x > 0$ , so  $x$  can't be negative.

Hence,  $x = 6$ .

(ii) Given quadratic equation is  $(x - 10) \left( \frac{1200}{x} + 2 \right) = 1260$

$$\Rightarrow (x - 10) \left( \frac{1200 + 2x}{x} \right) = 1260$$

$$\begin{aligned} \Rightarrow (x - 10)(1200 + 2x) &= 1260x \\ \Rightarrow 1200x + 2x^2 - 12000 - 20x &= 1260x \\ \Rightarrow 2x^2 - 12000 - 80x &= 0 \\ \Rightarrow x^2 - 40x - 6000 &= 0 \\ \Rightarrow x^2 - 100x + 60x - 6000 &= 0 \\ \Rightarrow (x - 100)(x - 60) &= 0 \\ \Rightarrow x = 100 \text{ or } x = -60 \end{aligned}$$

But as  $x < 0$ , so  $x$  can't be positive.

Hence,  $x = -60$ .