TOPIC QUADRATIC EQUATIONS:

LEVEL 1 (1 Mark each)

1. What will be the nature of roots of the quadratic equation: $2x^2 + 4x - 7 = 0$?
   Sol. $D = b^2 - 4ac$
   $$= 4^2 - 4 \times 2 \times (-7)$$
   $$= 16 + 56 = 76 > 0$$

   Hence, roots of quadratic equation are real and unequal.
2. Show that $x = -2$ is a solution of $3x^2 + 13x + 14 = 0$
   Sol. Put the value of x in the quadratic equation
   L.H.S = $3x^2 + 13x + 14$
   $$= 3(-2)^2 + 13(-2) + 14$$
   $12 - 26 + 14 = 0$ = R.H.S
   Hence, $x = -2$ is a solution of $3x^2 + 13x + 14 = 0$
3. Check if $(x + 1)^2 = 2(x - 3)$ is a quadratic or not?
   Sol. $(x + 1)^2 = 2(x - 3)$
   $$x^2 + 2x + 1 = 2x - 6$$
   $$x^2 + 7 = 0$$

Which is a quadratic.

4. Find the roots of the quadratic $2x^2 - 7x + 3 = 0$
   Sol. $2x^2 - 7x + 3 = 0$
   $2x^2 - 6x - x + 3 = 0$
   $2x(x - 3) - (x - 3) = 0$
   $(2x - 1)(x - 3) = 0$
   $$(2x - 1) = 0 \; 0r \; x - 3 = 0$$
   $$x = \frac{1}{2}, x = 3$$

LEVEL 2 (2 Marks each)

1. Find the value of p for which the quadratic equation $4x^2 + px + 3 = 0$ has equal roots.
   Sol. For equal roots $\quad D = 0$
   $$b^2 - 4ac = 0$$
   i.e., $p^2 - 4 \times 4 \times 3 = 0$
   $$p^2 - 48 = 0$$
   $$p^2 = 48$$
   $$p = \pm 4\sqrt{3}$$
2. Find two consecutive positive integers, sum of whose squares is 925.

Sol. Let two consecutive positive integers are x and x+1

A.T.Q

$$x^2 + (x+1)^2 = 925$$
$$x^2 + x^2 + 2x + 1 = 925$$
$$2x^2 + 2x + 1 = 925$$
$$2x^2 + 2x - 924 = 0$$
$$x^2 + x - 462 = 0$$
$$x^2 + 12x - 11x - 462 = 0$$
$$x(x+12) - 11(x+12) = 0$$
$$(x+12)(x-11) = 0$$
$$x = -12 \ \ 0r\ x = 11$$

$x = -12$is not possible

Therefore integers are 11 and 11+1 or 11 and 12.

3. Solve the quadratic $2x^2 - 7x + 3 = 0$ by using quadratic formula.

Sol. We have $2x^2 - 7x + 3 = 0$

Here $a = 2, b = -7 \ and \ c = 3$

Therefore $D = b^2 - 4ac$

$$D = (-7)^2 - 4 \times 2 \times 3$$
$$= 49 - 24 = 25 > 0$$

Therefore roots exist.

Now $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad x = \frac{-(-7) \pm \sqrt{25}}{2 \times 2}$

$$x = \frac{7 \pm 5}{4}$$
$$x = \frac{12}{4} \ or \ x = \frac{2}{4}$$
$$x = 3 \ or \ x = \frac{1}{2}$$

4. For what value of k, is 3 a root of the equation $2x^2 + x + k = 0$?

Sol. 3 is root of $2x^2 + x + k = 0$ then,

$2(3)^2 + 3 + k = 0$

18+$3 + k = 0$

21+$k = 0$.

K = - 21

LEVEL 3 (3 – Marks each)

1. Solve $4x^2 + 4\sqrt{3}x + 3 = 0$ by the method of completing squares.

   Sol. Given equation is $4x^2 + 4\sqrt{3}x + 3 = 0$

   Dividing both sides by 4

$$x^2 + \sqrt{3}x + \frac{3}{4} = 0$$

$$x^2 + \sqrt{3}x = -\frac{3}{4}$$

Adding square of half the coefficient of x to both sides

$$x^2 + \sqrt{3}x + \left(\frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\left(x + \frac{\sqrt{3}}{2}\right)^2 = 0$$

$$\left(x + \frac{\sqrt{3}}{2}\right)\left(x + \frac{\sqrt{3}}{2}\right) = 0$$

$$x = -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

Hence the roots of the equation $4x^2 + 4\sqrt{3}x + 3 = 0$ are $-\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$

2. Had Anita scored 10 more marks in her mathematics test out of 30 marks, 9 times these marks would have been the square of her actual marks. How many marks did she get in the test?

Sol. Let her actual marks be x

Therefore, $9(x+10) = x^2$

⇨ $x^2 - 9x - 90 = 0$

⇨ $x^2 - 15x + 6x - 90 = 0$

⇨ $x(x - 15) + 6(x - 15) = 0$

⇨ $(x + 6)(x - 15) = 0$

Therefore, $x = -6$ or $x = 15$

Since x is the marks obtained, $x \neq -6$. Therefore, $x = 15$.

So, Ajita got 15 marks in her mathematics test.

3. Solve for x ; $\frac{1}{2x-3} + \frac{1}{x-5} = 1, x \neq \frac{3}{2}, 5$

Sol.

$$\frac{1}{2x-3} + \frac{1}{x-5} = 1$$

$$\frac{(x-5)+(2x-3)}{(2x-3)(x-5)} = 1$$

$$\frac{3x-8}{2x^2-13x+15} = 1$$

$$2x^2 - 16x + 23 = 0$$

$$a = 2, b = -16, c = 23$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{16 \pm \sqrt{(-16)^2 - 4 \times 2 \times 23}}{2 \times 2}$$

$$= \frac{16 \pm \sqrt{72}}{4}$$

$$x = \frac{8 \pm 3\sqrt{2}}{2}$$

4. A cottage industry produces a certain number of articles in a day. It was observed on a particular day that the cost of production of each article war 3 more than the twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Solution: Let the number of articles be x

Therefore the cost of production = 3+2x

Total cost of production = Rs. 90

According to questions: $x(3 + 2x) = 90$

$$2x^2 + 3x - 90 = 0$$

$$2x^2 + 15x - 12x - 90 = 0$$

$$x(2x + 15) - 6(2x + 15) = 0$$

$$(2x + 15)(x - 6) = 0$$

$$x = -\frac{15}{2} \; or \; x = 6$$

$x = -\frac{15}{2}$is not possible so $x = 6$

Hence number of articles produced =6

And cost of production of each article is 15.

LEVEL 4 (4 –Marks each)

1. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol. Let the speed of the stream be *x* km/h.
Therefore, the speed of the boat upstream = (18 – *x*) km/h and the speed of the boat downstream = (18 + *x*) km/h.
The time taken to go upstream = $\frac{distance}{speed} = \frac{24}{18-x}$ hours.
Similarly, the time taken to go downstream =$\frac{24}{18-x}$hours.
According to the question,

$\frac{24}{18-x}$ - $\frac{24}{18+x}$ = 1

- ⇨ $24(18 + x) – 24(18 – x) = (18 – x)(18 + x)$
- ⇨ $324 + 24x - 324 + 24x = 324 – x^2$
- ⇨ $x^2 + 48x – 324 = 0$

Using the quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-48 \pm \sqrt{48^2 - 4 \times 1 \times (-324)}}{2 \times 1}$$

$$x = \frac{-48 \pm \sqrt{3600}}{2}$$

$$x = \frac{-48 \pm 60}{2}$$

$$x = \frac{-48 + 60}{2}, x = \frac{-48 - 60}{2}$$

$$x = 6, -54$$

Since x is the speed of stream it cannot be negative. So, we ignore the root $x = –54$.
Therefore, $x = 6$ gives the speed of the stream as 6 km/h.

2. A train travels at a certain average speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/h more than its original speed. If it takes 3 hours to complete the total journey, what is its original average speed?

Sol. Let its original average speed be x km/h. Therefore,

A.T.Q

$$\frac{63}{x} + \frac{72}{x+6} = 3$$

$$63(x + 6) + 72x = 3x(x + 6)$$

$$63x + 378 + 72x = 3x^2 + 18x$$

$$3x^2 - 117x - 378 = 0$$

$$3x^2 - 39x - 126 = 0$$

$$x^2 - 42x + 3x - 126 = 0$$

$$(x + 3)(x - 42) = 0$$

$$x = -3 \text{ or } x = 42$$

Since x is the average speed of the train, x cannot be negative. Therefore, $x = 42$
So, the original average speed of the train is 42 km/h.

3. A two digit number is such that the product of its digits is 18. When 63 is subtracted from the number the digits interchange their places. Find the number.

Sol.let the digit at tens place be x, then digit at unit place $=\frac{18}{x.}$.

Therefore number=10x+$\frac{18}{x}$

and the number obtained by interchanging the digit $=10\times\frac{18}{x}+x$

A.T.Q

$(10x+\frac{18}{x})-63=10\times\frac{18}{x}+x$

$(10x+\frac{18}{x})-(10\times\frac{18}{x}+x)=63$

$$10x+\frac{18}{x}-\frac{180}{x}-x=63$$
$$x^2-7x-18=0$$
$$x^2-9x+2x-18=0$$
$$(x-9)(x+2)=0$$
$$x=9,-2$$

Therefore $x=9$ (as a digit cannot be negative)

Hence the required number = $10\times 9+\frac{18}{9}=92$

4. A train takes 2 hours less for the journey of 300 km, if its speed is increased by 5 km/h from its usual speed. Find the usual speed of train.

Sol. Let the usual speed of train = x km/h

Therefore, time taken to cover 300 km = $\frac{300}{x}$ hours

When its speed is increased by 5 km/h, then time taken by the train tom cover the distance of 300 km = $\frac{300}{x+5}$ hours

According to the question,

$$\frac{300}{x}-\frac{300}{x+5}=2$$

$$\frac{300(x+5)-300x}{x(x+5)}=2$$
$$\frac{300x+1500-300x}{x^2+5x}=2$$
$$x^2+5x-750=0$$
$$(x-25)(x+30)=0$$
$$x=25\ or\ x=-30$$

$x=25$ (as speed cannot be negative)

Therefore speed of the train = 25 km/h