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Class 11 Conic Section

Miscellaneous

Q.1) A rod of length 12cm moves with its ends always touches the coordinate axis. Determine the equation of the locus of a point P on the rod which is 3cm from the end in contact with the $X - axis$.

Sol.1) In $\Delta PMA, \sin\theta = \frac{y}{3}$

$\sin^2\theta = \frac{y^2}{9}$ ……. (i)

In $\Delta PNB,\ \cos\theta = \frac{x}{9}$

$\cos^2\theta = \frac{x^2}{81}$ …….. (ii)

Adding (i) & (ii)

$\sin^2\theta + \cos^2\theta = \frac{y^2}{9} + \frac{x^2}{81} = 1$

$\Rightarrow \frac{x^2}{81} + \frac{y^2}{9} = 1$

clearly this equation represents the equation of the ellipse

∴ locus of point P is an ellipse.

Q.2) An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, where one vertex is at the vertex of the parabola. Find the length of the side of the triangle.

Sol.2) Let $AD = x$

∴ $AB = 2y$

And $AC = BC = 2y$ (equilateral triangle)

Equation of parabola is $y^2 = 4ax$ …… (i)

In ΔADC

Pythagoras : $4y^2 = x^2 + y^2$

$\Rightarrow 3y^2 = x^2$

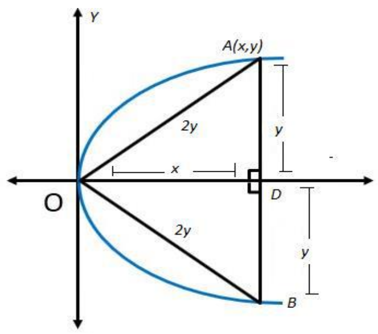
$\Rightarrow x = \sqrt{3}\,y$ put in equation (i)

We have, $y^2 = 4a(\sqrt{3}y)$

$y^2 = 4a\sqrt{3}y$

$\Rightarrow y = 4a\sqrt{3}$

Now side of $\Delta ABC = AB = 2y = 8a\sqrt{3}$ ans.



Q.3) A man running a race course notes that the sum of the distances from the two flag posts from him is always 10m and the distance between the flag posts is 8m. Find the equation of the paths traced by the man.

Sol.3) Distance between two foci $= SS' = 2ae$

$\Rightarrow 2ae = 8$

$\Rightarrow ae = 4$

Sum of focal distance $= SP + S'P = 2a$

$\Rightarrow 2a = 10$

$\Rightarrow a = 5$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{\frac{a^2 - b^2}{a^2}}$

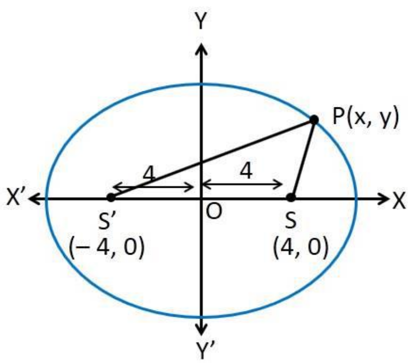
$\Rightarrow ae = \sqrt{a^2 - b^2}$

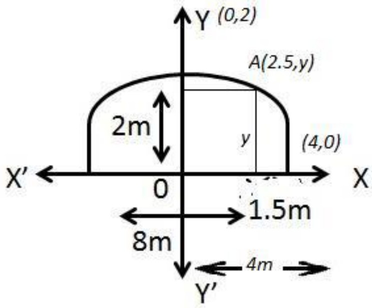
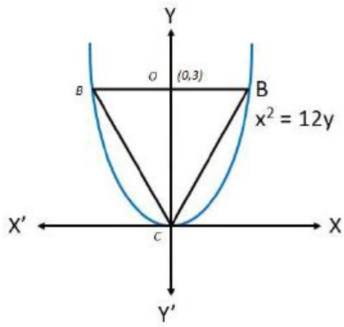
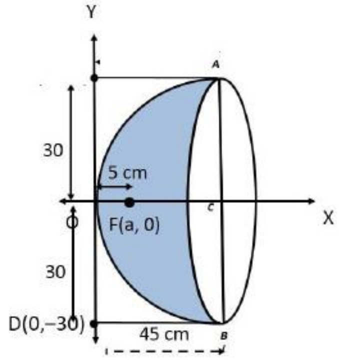
$\Rightarrow 4 = \sqrt{25 - b^2}$

$\Rightarrow 16 = 25 - b^2$

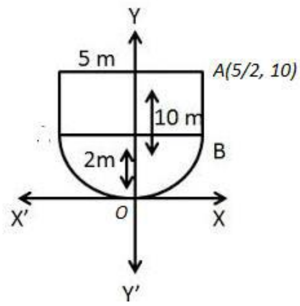
$\Rightarrow b^2 = 9$

Equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$



	$\Rightarrow \frac{x^2}{25} + \frac{y^2}{9} = 1$ ans.
Q.4)	An arc is in the form of a semi-ellipse. It is 8m wide and 2m high at the centre. Find the height of the arc at a point 1.5m from one end.
Sol.4)	Let the required $= y\ m$ From figure $a = 4$ and $b = 2$ $\therefore$ equation of ellipse is $\frac{x^2}{4^2} + \frac{y^2}{2^2} = 1$ $\Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1$ Now, $A(2.5, y)$ lies on it $\Rightarrow \frac{6.25}{16} + \frac{y^2}{4} = 1$ $\Rightarrow 6.25 + 4y^2 = 16$ $\Rightarrow 4y^2 = 16 - 6.25$ $\Rightarrow 4y^2 = 9.75$ $\Rightarrow y^2 = \frac{9.75}{4} = 2.437$ $\Rightarrow y = \sqrt{2.437} = 1.56\ m$ ans. 
Q.5)	Find the area of the riangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum.
Sol.5)	Equation of $x^2 = 12y$ Compare with $x^2 = 4ay$ $\Rightarrow 4a = 12$ $\Rightarrow a = 3$ $\therefore$ focus is $(0, a) = (0,3)$ $\therefore CD = 3$ (altitude of triangle) Now, latus rectum$= 4a = 4 \times 3 = 12$ (Base of triangle) Area of the triangle ΔABC $= \frac{1}{2} \times 12 \times 3$ $= 18\ square\ units$ ans. 
Q.6)	The focus of a parabolic mirror is at a distance of 5cm from its vertex. If the mirror is 45cm deep. Find the distance AB(diameter).
Sol.6)	Let $AC = y$ $\therefore AB = 2y$ Equation of parabola is $y^2 = 4ax$ Focus is at $(5,0)$ Compare with $(a, 0)$ We get $a = 5$ $\therefore$ equation of parabola becomes $y^2 = 4(5)x$ $\Rightarrow y^2 = 2ax$ Now $A(45, y)$ lies on parabola $\Rightarrow y^2 = 20 \times 45$ $\Rightarrow y^2 = 900$ $\Rightarrow y = 30$ $\therefore$ required length (diameter) $= 2y = 60cm$ ans. 
Q.7)	An arc is in the form of a parabola with its axis vertical. The arc is 10m high and 5m wide at the base. How wide is it 2m from the vertex of the parabola?
Sol.7)	Let required width $= 2x$ Let equation of parabola is $x^2 = 4ay$ .... (i)

$A\left(\frac{5}{2}, 10\right)$ lies on it
$\Rightarrow \frac{25}{4} = 40a$
$\Rightarrow a = \frac{25}{160}$
Put in equation (i)
$\therefore x^2 = 4\left(\frac{25}{160}\right)y$
$\Rightarrow x^2 = \frac{25}{40}y$
Now $B(x, 2)$ lies on it
$\Rightarrow B(x, 2)$ lies on it
$\Rightarrow x^2 = \frac{25}{40} \times 2$
$\Rightarrow x^2 = \frac{25}{20} = \frac{5}{4} = 1.25$
$\Rightarrow x = 1.1$
$\therefore$ required width $= 2x = 2(1.1) = 2.2m$ ans.



➢ SHIFTING PORABOLA

Q.8) Given equation of parabola $y^2 - 8y - x + 19 = 0$.
Find vertex, focus, axis, directrix, latus rectum.

Sol.8) Given, $y^2 - 8y - x + 19 = 0$
$\Rightarrow y^2 - 8y = x - 19$
$\Rightarrow (y-4)^2 - 16 = x - 19$
$\Rightarrow (y-4)^2 = (x-3)$
Let $y - 4 = Y$ and $x - 3 = X$
$\Rightarrow y = Y + 4$ and $x = X + 3$
$\therefore$ equation becomes $Y^2 = X$
Comparing this equation with $Y^2 = 4aX$
We have, $ya = 1 \Rightarrow$ a $= \frac{1}{4}$
i) Vertex with respect to new axis $(X, Y) = (0,0)$
vertex with respect to old axis $(x, y) = (3,4)$
ii) focus with respect to new axis $= (a, 0) = \left(\frac{1}{4}, 0\right)$
focus with respect to old axis $= \left(\frac{13}{4}, 4\right)$
iii) directrix with respect to new axis $X = -a$
$\Rightarrow X = -\frac{1}{4}$
directrix with respect to old axis $x = X + 3$
$\Rightarrow x = -\frac{1}{4} + 3$
$\Rightarrow x = \frac{11}{4}$
iv) Latusrectum $4a = 4 \times \frac{1}{4} = 1$
v) Axis with respect new axis $Y = 0$
Axis with respect to old axis $y = Y + 4$
$\Rightarrow y = 0 + 4 = 4$

Q.9) Find vertex, focus, directrix and axis of the parabola $4y^2 + 12x - 12y + 39 = 0$.

Sol.9) Given, $4y^2 + 12x - 12y + 39 = 0$
$\Rightarrow 4y^2 - 12y = -12x - 39$
$\Rightarrow 4(y^2 - 3y) = -12x - 39$
$\Rightarrow 4\left[\left(y - \frac{3}{2}\right)^2 - \frac{9}{4}\right] = -12x - 39$

	$\Rightarrow 4\left(y-\frac{3}{2}\right)^2 - 9 = -12x - 39$ $\Rightarrow 4\left(y-\frac{3}{2}\right)^2 = -12x - 30$ $\Rightarrow 4\left(y-\frac{3}{2}\right)^2 = -12\left(x+\frac{5}{2}\right)$ $\Rightarrow \left(y-\frac{3}{2}\right)^2 = -3\left(x+\frac{5}{2}\right)$ Now let $x+\frac{5}{2} = X$ and $y-\frac{3}{2} = Y$ $\therefore x = X-\frac{5}{2}$ and $y = Y+\frac{3}{2}$ $\therefore$ equation becomes $Y^2 = -3x$ Comparing this equation with $Y^2 = -4ax$ We have $4a = 3 \Rightarrow a = \frac{3}{4}$ i) Vertex with respect to new axis = $(0,0)$ vertex with respect to old axis = $\left(-\frac{5}{2},\frac{3}{2}\right)$ ii) focus with respect to new axis = $(-a,a) = \left(-\frac{3}{4},0\right)$ focus with respect to old axis = $\left(-\frac{3}{4}-\frac{5}{2},0+\frac{3}{2}\right) = \left(-\frac{13}{4},\frac{3}{2}\right)$ iii) directrix with respect to new axis $X = a \Rightarrow \frac{3}{4}$ directrix with respect to $x = X-\frac{5}{2}$ $\Rightarrow x = \frac{3}{4}-\frac{5}{2} = -\frac{7}{4}$ iv) axis with respect to new axis $y = Y+\frac{3}{2}$ $\Rightarrow y = 0+\frac{3}{2} = \frac{3}{2}$
➢	**SHIFTING ELLIPSE**
Q.10)	Find e, centre, vertices, foci, minor axis, major axis, directrix and latus rectum of the ellipse $25x^2+9y^2-150x-90y+225=0$.
Sol.10)	Given, $25x^2+9y^2-150x-90y+225=0$ $\Rightarrow 25x^2-150x+9y^2-90y+225=0$ $\Rightarrow 25[x^2-6x]+9(y^2-10y)+225=0$ $\Rightarrow 25[(x-3)^2-9]+9[(y-5)^2-25]+225=0$ $\Rightarrow 25(x-3)^2-225+9((y-5)^2-225+225=0$ $\Rightarrow 25(x-3)^2+9(y-5)^2=225$ $\Rightarrow \frac{25(x-3)^2}{225}+\frac{9(y-5)^2}{225}=1$ $\Rightarrow \frac{(x-3)^2}{9}+\frac{(y-5)^2}{25}=1$ Let $x-3=X$ and $y-5=Y$ $\Rightarrow x = X+3$ and $y = Y+5$ $\therefore$ equation becomes $\frac{x^2}{9}+\frac{y^2}{25}=1$ Here $a=3$ and $b=5$ After compare the equation with $\frac{X^2}{a^2}+\frac{Y^2}{b^2}=0$ i) eccentricity $e = \sqrt{1-\frac{a^2}{b^2}} = \sqrt{1-\frac{9}{25}}$ $e = \frac{4}{5}$ ii) centre with respect to new axis $X,Y = (0,0)$

centre with respect to old axis $(0+3, 0+5)$ $(x, y) = (3,5)$

iii) vertices with respect to new axis $= (0, \pm b) = (0, \pm 5)$

vertices with respect to old axis $= (0+3, 5+5)$ and $(+3, -5+5)$

$= (3,10)$ and $(3,0)$

iv) foci with respect to new axis $(0, \pm b) = (0, \pm 4)$

foci with respect to old axis $= (0+3, 4+5)$ and $(0+3, -4+5)$

$= (3,9)$ and $(3,1)$

v) directrix with respect to new axis $Y = \pm \frac{b}{e}$

$\Rightarrow Y = \pm \frac{5}{\frac{4}{5}} = \pm \frac{25}{4}$

Directrix with respect to old axis $y = Y + 5$

$\Rightarrow y = \frac{25}{4} + 5$ and $y = \frac{-25}{4} + 5$

$\Rightarrow y = \frac{45}{4}$ and $y = \frac{-5}{4}$

vi) major axis length $2b = 2(5) = 10$

vii) minor axis length $2a = 2(3) = 6$

viii) latus rectum $\frac{2a^2}{b} = \frac{2(9)}{5} = \frac{18}{5}$