Pair of Linear Equations in Two Variables

(Key Points)

- An equation of the form ax + by + c = 0, where a, b, c are real nos. ($a \neq 0, b \neq 0$) i.e ($a^2+b^2 \neq 0$) is called a linear equation in two variables x and y.

  Ex : (i) $x - 5y + 2 = 0$

  (ii) $\frac{3}{2}x - y = 1$

- The general form for a pair of linear equations in two variables x and y is

  $a_1x + b_1y + c_1 = 0$

  $a_2x + b_2y + c_2 = 0$

  Where $a_1, b_1, c_1, a_2, b_2, c_2$ are all real nos and $a_1 \neq 0, b_1 \neq 0, a_2 \neq 0, b_2 \neq 0$.

  Examples: $x + 3y - 6 = 0$

  $2x - 3y - 12 = 0$

- Graphical representation of a pair of linear equations in two variables:

  $a_1x + b_1y + c_1 = 0$

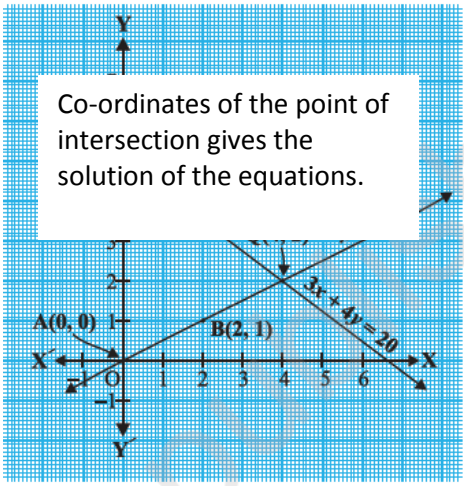
  $a_2x + b_2y + c_2 = 0$

  (i) Will represent intersecting lines if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

  I.e. unique solution. And these types of equations are called consistent pair of linear equations.

  Ex: x – 2y = 0

  3x + 4y – 20 = 0

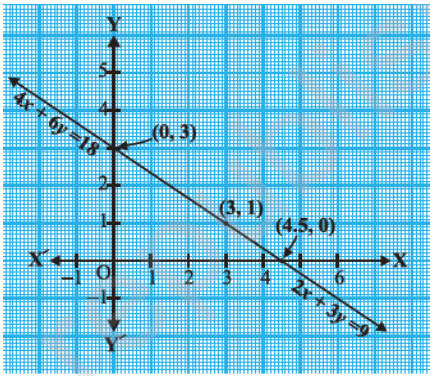


  (ii) will represent overlapping or coincident lines if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

  i.e. Infinitely many solutions, consistent or dependent pair of linear equations

  Ex: 2x + 3y – 9 = 0

  4x + 6y – 18 = 0



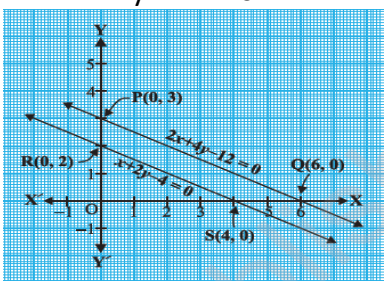
The graph is Coincident lines,

(iii) will represent parallel lines if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e. no solution and called inconsistent pair of linear equations.

Ex: x + 2y – 4 = 0

2x + 4y – 12 = 0



Parallel lines, no solution.

- Algebraic methods of solving a pair of linear equations:

(i) Substitution method
(ii) Elimination Method
(iii) Cross multiplication method

Level - I

1. Find the value of 'a' so that the point(2,9) lies on the line represented by ax-3y=5
2. Find the value of k so that the lines 2x – 3y = 9 and kx-9y =18 will be parallel.
3. Find the value of k for which x + 2y =5, 3x+ky+15=0 is inconsistent
4. Check whether given pair of lines is consistent or not 5x – 1 = 2y, y = $\frac{-1}{2} + \frac{5}{2}x$
5. Determine the value of 'a' if the system of linear equations 3x+2y -4 =0 and ax – y – 3 = 0 will represent intersecting lines.
6. Write any one equation of the line which is parallel to $\sqrt{2}x - \sqrt{3}y = 5$
7. Find the point of intersection of line -3x + 7y =3 with x-axis
8. For what value of k the following pair has infinite number of solutions.

   (k-3)x + 3y = k

   k(x+y)=12
9. Write the condition sothat $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ have unique solution.

Level - II

1. 5 pencils and 7pens together cost Rs. 50 whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.
2. Solve the equations:

   3x – y = 3

   7x + 2y = 20
3. Find the fraction which becomes to 2/3 when the numerator is increased by 2 and equal to 4/7 when the denominator is increased by 4
4. Solve the equation:

   px + qy = p – q

   qx – py = p + q
5. Solve the equation using the method of substitution:

   $3x - 5y = -1$

   $x - y = -1$
6. Solve the equations:

   $\frac{1}{2x} - \frac{1}{y} = -1$

   $\frac{1}{x} + \frac{1}{2y} = 8$ Where, $x \neq 0, y \neq 0$
7. Solve the equations by using the method of cross multiplication:

   $x + y = 7$

   5x + 12y =7

Level - III

1. Draw the graph of the equations

   4x – y = 4

   4x + y = 12

   Determine the vertices of the triangle formed by the lines representing these equations and the x-axis. Shade the triangular region so formed
2. Solve Graphically

   x – y = -1 and

   3x + 2y = 12

Calculate the area bounded by these lines and the x- axis,

3. Solve :- for u & v

   4u – v = 14uv

   3u + 2v = 16uv where u≠0, v≠ 0

4. Ritu can row downstream 20 km in 2 hr , and upstream 4 km in 2 hr . Find her speed of rowing in still water and the speed of the current. (HOTS)

5. In a ΔABC ,$\angle C$ = 3∠B = 2 (∠A +∠B) find the these angle. (HOTS)

6. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by 1 man alone and that by one boy alone to finish the work. (HOTS)

7. Find the value of K for which the system of linear equations 2x+5y = 3, (k +1)x + 2(k + 2) y = 2K will have infinite number of solutions. (HOTS)

SELF EVALUTION

1. Solve for x and y:

   x + y = a + b

   ax – by= $a^2 - b^2$

2. For what value of k will the equation x +5y-7=0 and 4x +20y +k=0 represent coincident lines?

3. Solve graphically: 3x +y +1=0

   2x -3y +8=0

4. The sum of digits of a two digit number is 9. If 27is subtracted from the number, the digits are reversed. Find the number.

5. Draw the graph of x + 2y – 7 =0 and 2x – y -4 = 0. Shade the area bounded by these lines and Y-axis.

6. Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row there would be 3 rows more. Find the number of the students in the class.

7. A man travels 370 km partly by train and remaining by car. If he covers 250 km by train and the rest by the car it takes him 4 hours, but if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.
8. Given linear equation 2x +3y-8=0, write another linear equation such that the geometrical representation of the pair so formed is (i) intersecting lines, (ii) Parallel Lines.
9. Solve for x and y.

   $(a-b)x + (a+b)y = a^2 - 2ab - b^2$

   $(a+b)(x+y) = a^2+b^2$ **(CBSE 2004, '07C, '08)**

10. The sum of two numbers is 8 and the sum of their reciprocal is 8/15. Find the numbers.

(CBSE 2009)

Value Based Questions

Q1. The owner of a taxi cab company decides to run all the cars he has on CNG fuel instead of petrol/diesel. The car hire charges in city comprises of fixed charges together with the charge for the distance covered. For a journey of 12km, the charge paid Rs.89 and for a journey of 20 km, the charge paid is Rs. 145.

i. What will a person have to pay for travelling a distance of 30 km?

ii. Which concept has been used to find it?

iii. Which values of the owner have been depicted here?

Q2.Riya decides to use public transport to cover a distance of 300 km. She travels this distance partly by train and remaining by bus. She takes 4 hours if she travels 60km by bus and the remaining by train. If she travels 100 km by bus and the remaining by train, she takes 10 minutes more.

i. Find speed of train and bus separately.

ii. Which concept has been used to solve the above problem?

iii. Which values of Riya have been depicted here?

ANSWER

LEVEL-I

Q1.a= 16
Q2.k= 6
Q3.k= 6
Q4.Consistent
Q5. $a \neq \frac{-3}{2}$
Q6.$5\sqrt{2}x - 5\sqrt{3}y = 5\sqrt{5}$ (May be another solution also)
Q7.(-1, 0)
Q8.k= 6
Q9.$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

LEVEL-II

Q1.: Cost of one pencil = Rs. 3

Cost of one pen = Rs. 5

Q2.x=2, y=3
Q3.28/45
Q4.x = 1, y = -1
Q5. $x = -2, \quad y = -1$

Q6.$x = \frac{1}{6}, y = \frac{1}{4}$

Q7.$x = 11, y = -4$

LEVEL-III

Q1.(2,4)(1,0)(3,0)
Q2.x = 2, y = 3 and area = 7.5 unit ^{2}
Q3.u = ½ , v = ¼
Q4. Speed of the rowing in still water = 6 km/hr

Speed of the current = 4 km/hr .

Q5. $\angle A = 20^0, \angle B = 40^0, \angle C = 120^0$.

Q6.: One man can finish work in 140 days.

One boy can finish work in 280 days.

Q7.K = 3

SELF EVALUATION

Q1.X=a,y=b
Q2.K=-28
Q3.X= -1, y=2
Q4.63
Q6.60

Q7.Speed of the train=100km/h, speed of the car=80km/h
Q8.(i) 4x-3y-8 =0 (may be another equation also)
(ii) 4x+6y+16 =0 (may be another equation also)
Q9.X= a+b,y = -2ab/(a+b)
Q10.3,5

VALUE BASED QUESTIONS

Q1.(i)Rs.215,(ii)A pair of linear equations in two variables has been used to find it.

(iii) Awareness of environment.

Q2. (i) The speed of the train = 80 km/h, the speed of the bus = 60km/h

(ii) A pair of linear equations in two variables has been used.

(iii)Controlling the pollution of the environment.