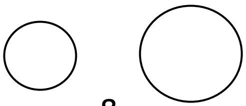
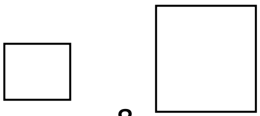
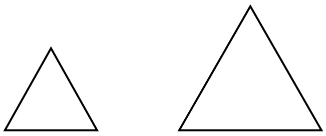
Triangles

Key Points

Similar Figures: Two figures having similar shapes (size may or may not same), called Similar figures.

Examples: (a)  (b)  (c) 

A pair of Circles — A pair of squares — A pair of Equ. Triangles

- Pairs of all regular polygons, containing equal number of sides are examples of Similar Figures.
- **Similar Triangles:** Two Triangles are said to be similar if
  - (a) Their corresponding angles are equal (also called Equiangular Triangles)
  - (b) Ratio of their corresponding sides are equal/proportional
- All congruent figures are similar but similar figures may /may not congruent
- Conditions for similarity of two Triangles

  (a) AAA criterion/A-A corollary

  (b) SAS similarity criterion

  (c) SSS similarity criterion (where 'S' stands for ratio of corresponding sides of two Triangles)

Important Theorems of the topicTriangles

(a) Basic Proportionality Theorem (B.P.T.)/Thale's Theorem

(b) Converse of B.P.T.

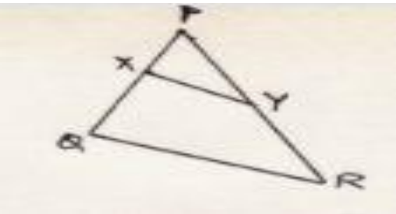
(c) Area related theorem of Similar Triangles

(d) Pythagoras Theorem

(e) Converse of Pythagoras Theorem

Level I

(1) In the figure XY ∕∕ QR , PQ/ XQ = 7/3 and PR =6.3cm then find YR



(2) If ΔABC ~ Δ DEF and their areas be $64cm^2$& $121cm^2$ respectively , then find BC if EF =15.4 cm

(3) ABC is an isosceles Δ ,right angled at C then prove that $AB^2 = 2AC^2$

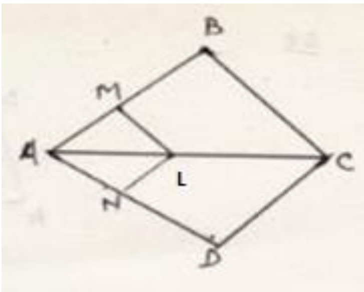
(4) If ΔABC ~ Δ DEF, ∟A=46^0, ∟E= 62^0 then the measure of ∟C=72^0. Is it true? Give reason.

(5) The ratio of the corresponding sides of two similar triangles is 16:25 then find the ratio of their perimeters.

(6) A man goes 24 km in due east and then He goes 10 km in due north. How far is He from the starting Point?

(7) The length of the diagonals of a rhombus is 16cm & 12cm respectively then find the perimeter of the rhombus.

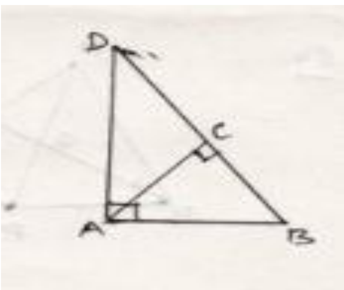
(8) In the figure LM ∕∕CB and LN ∕∕ CD then prove that AM/AB = AN /AD



(9) Which one is the sides of a right angled triangles among the following (a) 6cm,8cm & 11cm (b) 3cm,4cm & 6cm (c) 5cm , 12cm & 13cm

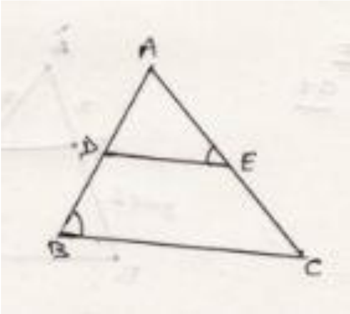
Level II

(1) In the figure ABD is a triangle right angled at A and AC is perpendicular to BD then show that AC^2= BC x DC

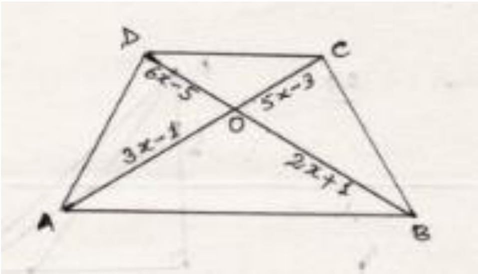


(2) Two poles of height 10m & 15 m stand vertically on a plane ground. If the distance between their feet is 5√3m then find the distance between their tops.

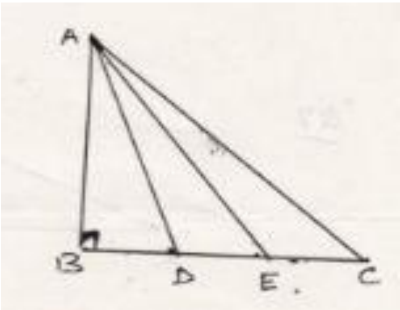
(3) D & E are the points on the sides AB & AC of ΔABC, as shown in the figure. If ∟B = ∟AED then show that ΔABC ~ΔAED



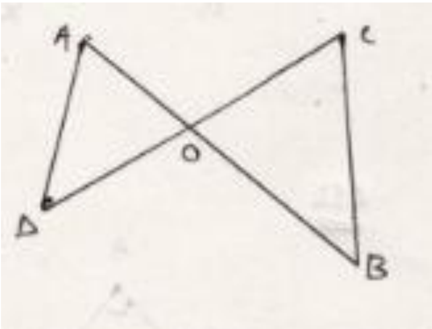
(4) In the adjoining figure AB ⁄⁄ DC and diagonal AC & BD intersect at point O. If AO = (3x-1)cm , OB= (2x+1)cm, OC=(5x-3)cm and OD=(6x-5)cm then find the value of x.



(5) In the figure D &E trisect BC. Prove that $8AE^2= 3AC^2+ 5AD^2$

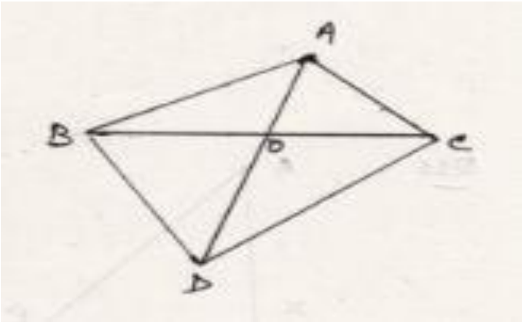


(6) In the figure OA/OC = OD /OB then prove that ∟A= ∟C



(7) Using converse of B.P.T. prove that the line joining the mid points of any two sides of a triangle is parallel to the third side of the triangle.

(8) In the given figure ΔABC &Δ DBC are on the same base BC . if AD intersect BC at O then prove that ar(ΔABC)/ar(Δ DBC) = AO/DO



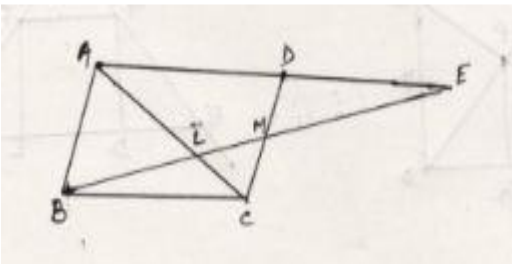
Level III

(1) A point O is in the interior of a rectangle ABCD, is joined with each of the vertices A, B, C & D. Prove that $OA^2 + OC^2 = OB^2 + OD^2$

(2) In an equilateral triangle ABC, D is a point on the base BC such that BD= 1/3 BC ,then show that $9AD^2 = 7AB^2$

(3) Prove that in a rhombus, sum of squares of the sides is equal to the sum of the squares of its diagonals

(4) In the adjoining figure ABCD is a parallelogram. Through the midpoint M of the side CD, a line is drawn which cuts diagonal AC at L and AD produced at E. Prove that EL =2BL



(5) ABC & DBC are two triangles on the same base BC and on the same side of BC with ∟A = ∟D $=90^0$. If CA & BD meet each other at E then show that AE x EC = BE x ED

(6) ABC is a Triangle, right angle at C and p is the length of the perpendicular drawn from C to AB. By expressing the area of the triangle in two ways show that (i) pc =ab (ii) $1/p^2 = 1/a^2 + 1/b^2$

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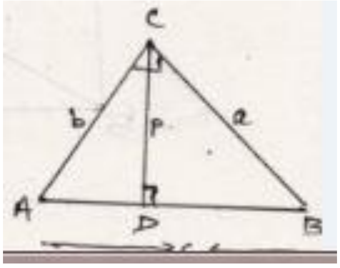
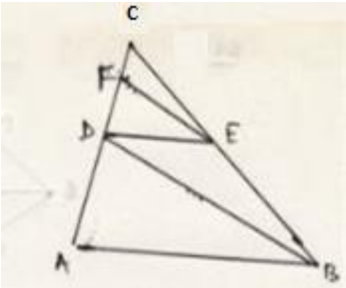


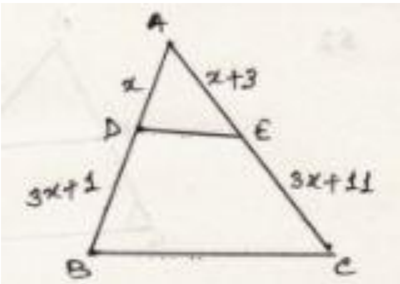
(7) Prove that the ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides.

(8) In the figure AB|| DE and BD|| EF. Prove that DC^2= CF x AC



Self-Evaluation Questions including Board Questions &Value Based Questions

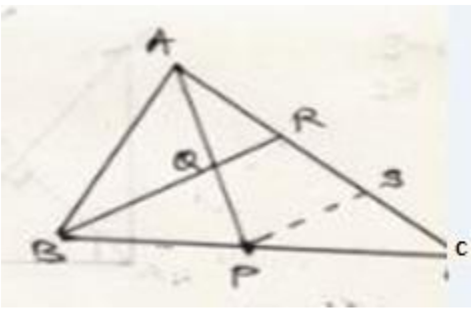
(1) Find the value of x for which DE ||BC in the adjoining figure



(2) In an equilateral triangle prove that three times the square of one side is equal to four times the square of one of its altitude.

(3) The perpendicular from A on the side BC of a triangle ABC intersect BC at D such that DB = 3CD. Prove that $2AB^2= 2AC^2+ BC^2$

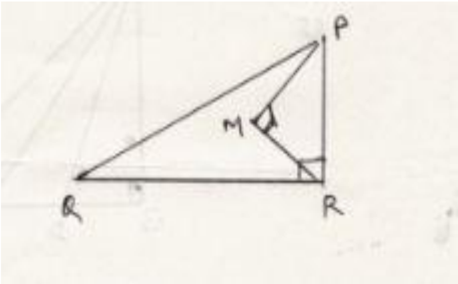
(4) In the adjoining figure P is the midpoint of BC and Q is the midpoint of AP. If BQ when produced meets AC at R ,then prove that RA = 1/3 CA



(5) BL and CM are medians of triangle ABC , right angled at A then prove that $4(BL^2+CM^2)= 5BC^2$

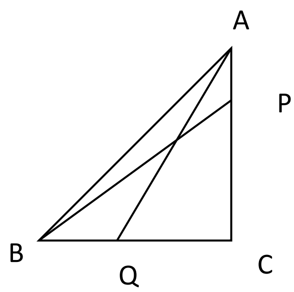
(6) In ΔABC if AB =6√3cm , AC =12cm and BC=6cm then show that ∟B =90^0

(7) In the adjoining figure ∠QRP =90^0, ∠PMR=90^0,QR =26cm, PM= 8cm and MR =6cm then find the area of ΔPQR

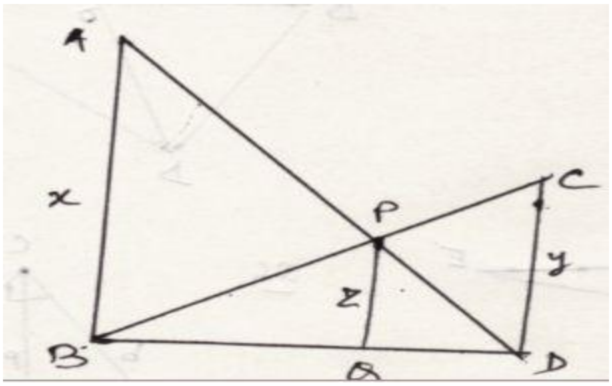


(8) If the ratio of the corresponding sides of two similar triangles is 2:3 then find the ratio of their corresponding altitudes.

(9) In the adjoining figure ABC is a Δ right angled at C. P& Q are the points on the sides CA & CB respectively which divides these sides in the ratio 2:1, then prove that $9(AQ^2 + BP^2) = 13\ AB^2$

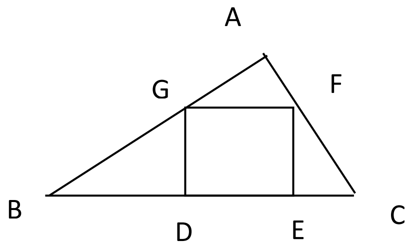


(10) In the adjoining figure AB || PQ ||CD, AB =x unit, CD= y unit & PQ = z unit then prove that 1/x +1/ y = 1/z



(11)State and prove Pythagoras theorem. Using this theorem find the distance between the tops of two vertical poles of height 12m & 18m respectively fixed at a distance of 8m apart from each other.

(12) in the adjoining figure DEFG is a square & ∟BAC= 90^0 then prove that (a) ΔAGF ~ Δ DBG (B) Δ AGF ~Δ EFC (C) Δ DBG ~Δ EFC (D)DE^2 = BD X EC



(13) A man steadily goes 4 m due east and then 3m due north .Find

(a) Distance from initial point to last point.

(b) What mathematical concept is used in this problem?

(c) What is its value?

Solutions

Level I

(1) By B.P.T. PQ/XQ=PR/YR⇨ 7/3=6.3/YR ⇨ YR= 3x6.3/7=2.7

So YR=2.7cm

(2) By theorem Ar of ΔABC/Ar of ΔDEF= $BC^2/15.4^2$

⇨$64/121 = BC^2/15.4^2$⇨solving BC = 11.2 cm

(3) By Pythagoras theorem $AB^2= AC^2+BC^2$ ⇨$AB^2= AC^2+AC^2$(given that AC=BC)

So $AB^2=2AC^2$

(4) ΔABC~ΔDEF ⇨ ∠A=∠D=46^0 , ∠B =∠ E=62^0 so ∠C =180-(46+62)=72^0

So it is true.

(5) Let ΔABC~ΔDEF

then AB/DE= BC/EF=AC/DF= perimeter of ΔABC/Perimeter of ΔDEF

⇨AB/DE=perimeter of ΔABC/Perimeter of ΔDEF

So perimeter of ΔABC/Perimeter of ΔDEF=16:25

(6) By Pythagoras theorem , Distance =$\sqrt{24^2+10^2}$

On Solving , distance =26km

(7) In ΔAOD, by Pythagoras theorem AD=$\sqrt{6^2+8^2}$

⇨AD= 10cm

So perimeter of Rhombus = 4x10cm

= 40cm

(8) In ΔABC ,LM//BC so by BPT AM/AB=AL/AC------(i)

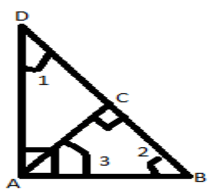
Similarly in ΔACD , LN//DC , so by BPT AN/AD = AL/AC-----------(ii)

Comparing results I &ii we get AM/AB= AN/AD

Using Pythagoras thermo ,finding the value of p^2+b^2&h^2 separately in each case , it comes equal in case of c where p^2+b^2 comes equal to h^2

So sides given in question c is the sides of right triangle

Level II



(1)In ΔABDABC ∠2+∠3=90^0

⇨∠1+∠2=∠2+∠3

⇨ ∠1=∠3

ΔACD~ΔBCA

⇨AC/BC= CD/AC

So AC^2=BC x CD

(2)Using Pythagoras theorem

Distance between their tops = $\sqrt{5^2+(5\sqrt{3})^2}$

$\sqrt{25 + 75}$

Distance between their tops= 10m

(3)In ΔAED&ΔABC

∠AED=∠ABC(given)

∠A=∠A(common)

By AA corollaryΔ ABC ~ ΔAED

(4)Diagonals of a trapezium divide each proportionally

So AO/OC =BO/OD

3x-1/5x-3= 2x+1/6x-5

⇨$8x^2-20x+8=0$

Solving we get x=2 &1/2(na)

So x=2

(5) BD=DE=EC =P(let)

BE=2P &BC=3P

In Rt ΔABD ,$AD^2= AB^2+BD^2$

$=AB^2+p^2$

In Rt ΔABE, $AE^2= AB^2+BE^2$

$=AB^2+(2p)^2$

$=AB^2+4p^2$

In Rt ΔABC,$AC^2=AB^2+BC^2$

$=AB^2+(3p)^2$

$=AB^2+9p^2$

Now taking RHS $3AC^2+5AD^2$

=3(AB2+9p2)+5(AB2+p2)

$=8AB^2+32p^2$

$=8(AB^2+4p^2)$

$=8AE^2$

=LHS

(6)OA/OC=OD/OB(given)

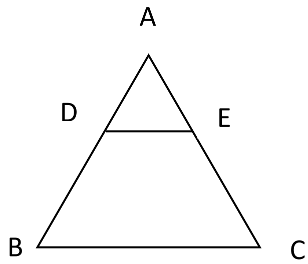
⇨OA/OD=OC/OB

&∠AOD=∠BOC(v.o.∠^{s})

By SAS similarity condition ΔAOD~ΔCOB

⇨∠A=∠C

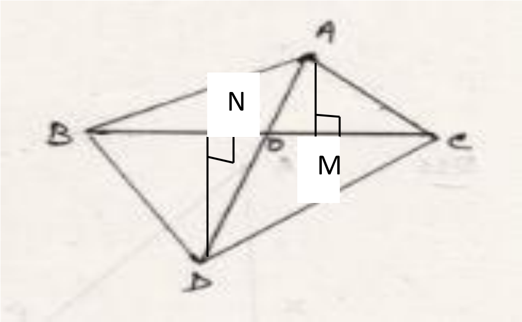
(7)



Given that AD/DE=1 &AE/EC=1(as D &E are mid points of the sides AB & AC)

⇨AD/DB =AE/EC

By converse of BPT DE//BC

(8) 

We draw perpendiculars AM & DN as shown .ΔDON~ΔAOM(by AA corollary)

DN/AM =OD/OA⇨AM/DN=OA/OD-------(i)

Ar of ΔABC/Ar of ΔDBC =(1/2xBCx AM)/(1/2x BC x DN)

=AM/DN

Ar of ΔABC/Ar of ΔDBC =AO/OD(from (i))

Level III

(1) We draw PQ ||BC through Pt. O⇨BPQC & APQD are rectangles.

In Rt ΔOPB , by Pythagoras theorem $OB^2=BP^2+OP^2$-------(i)

In Rt ΔOQD ,$OD^2=OQ^2+DQ^2$----------(ii)

In Rt ΔOQC ,$OC^2=OQ^2+CQ^2$--------(iii)

In Rt ΔOAP, $OA^2=AP^2+OP^2$-------------(iv)

On adding (i) &(ii)

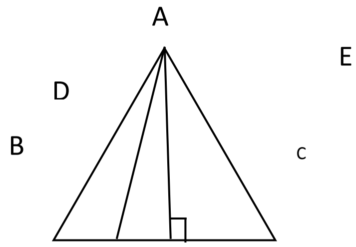
$OB^2+OD^2=BP^2+OP^2+OQ^2+PQ^2$

$=CQ^2+OP^2+OQ^2+AP^2$(BP=CQ & DA =AP)

$=CQ^2+OQ^2+OP^2+AP^2$

So OB^2+OD^2 $=OC^2+OD^2$

(2)



We draw AE perpendicular to BC & AD is joined.

Then BD = BC/3 , DC =2BC/3 & BE=EC =BC/2

In Rt. ΔADE,$AD^2=AE^2+DE^2$

$=AE^2+(BE-BD)^2$

$=AE^2+BE^2+BD^2-2.BE.BD$

$= AB^2+ (BC/3)^2-2.BC/2.BC/3$

$=AB^2+BC^2/9-BC^2/3$

$=(9AB^2+BC^2-3BC^2)/9$

$-=(9AB^2+AB^2-3AB^2)/9$(Given AB=BC=AC)

$=7AB^2/9$

⇨$9AD^2=7AB^2$

(3)In Rt.ΔAOB,$AB^2=OA^2+OB^2$

$=(AC/2)^2+(BD/2)^2$

$4AB^2= AC^2+BD^2$---------------(I)

Similarly $4BC^2=AC^2+BD^2$----------------(II)

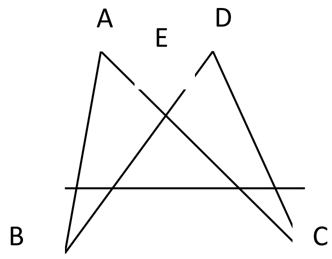
$4CD^2= AC^2+BD^2$--------------(III)

$4AD^2=AC^2+BD^2$-------------(IV)

Adding these results $4(AB^2+BC^2+CD^2+AD^2) =4(AC^2+BD^2)$

⇨$(AB^2+BC^2+CD^2+AD^2) =(AC^2+BD^2)$

(4)ΔBMC ≅ΔEDM(by ASA criterion)
⇨by cpct DE=BC & AD =BC (opp. sides of //gm)
Adding above results AD+DE=BC+BC
⇨AE =2BC
Now ΔAEL~ΔCBL (By AA corollary)
EL/BL=AE/BC⇨EL/BL =2BC/BC ⇨EL =2BL

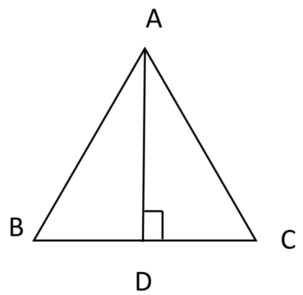


(5)ΔAEB~ΔDEC(AA corollary)
AE/DE=EB/EC
⇨AE X EC= BE X ED

(6)Ar of ΔABC=1/2x AB X DC
=1/2 X c Xp
= pc/2
Again Ar of ΔABC= ½ x AC X BC
=1/2 x b x a
= ab/2
Comparing above two areas
ab/2= pc/2
⇨ab=pc
Now in Rt ΔABC,$AB^2=BC^2+AC_2$
$c^2=a^2+b^2$
$(ab/p)^2= a^2+b^2$)(ab=pc⇨c=ab/p)
$a^2b^2/p^2=a^2+b^2$
$1/p^2=a^2+b^2/a^2b^2$
$1/p^2= 1/a^2 +1/b^2$
(7) Theorem question, as proved
(8) In ΔABC,AB //DE , by BPT AC/DC BC/CE--------(i)
In ΔDBC, EF//BD, by BPT DC/CF = BC/EC -----------(ii)
Comparing (i) &(ii) AC/DC=DC/CF
⇨DC^2=AC X CF

Self-Evaluation Questions

(1) A/Q AD/DB = AE/EC (by BPT)
⇨x/3x+1= x+3/3x+11
⇨3x2+11=3x2+9x+x+3
So x=3
(2)



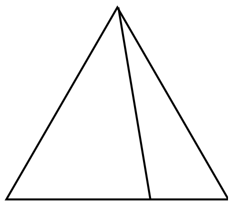
In ΔABD,$AB^2=AD^2+BD^2$
$= AD^2+(BC/2)^2$(AB=BC=AC)

-= $AD^2+AB^2/4$

$4AB^2=4AD^2+AB^2$

$4AB^2-AB^2= 4AD^2$

$3AB^2=4AD^2$



(3), BC =4CD ⇨CD = BC/4

⇨ BD =3CD= 3BC/4 ---------(i)

In ΔABD ,$AB^2=AD^2+BD^2$-------(ii)

In ΔACD, $AC^2= AD^2+CD^2$------(iii)

Now $AB^2-AC^2= BD^2=CD^2$

= $9BC^2/16- BC^2/16= BC^2/2$

$2(AB^2-AC^2) =BC^2$

$2AB^2=2AC^2+BC^2$

(4) we draw PS||BR

In triangle RBC, P is the mid point of BC and PS||BR

RS=CS [Mid point theorem] ……………………..(1)

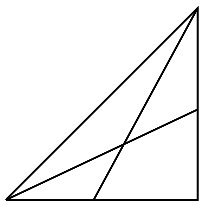
In Δ APS, PS||BR ie PS||QR and Q is the mid point of AP

So AR=RS……………………….[II](Mid point theorem)

From results (I)&(II) AR=RS=CS

So AR =1/3AC

(5)



In ΔABL $BL^2=AB^2+AL^2$

$4BL^2=4AB^2+4AL^2$

$=4AB^2+(2AL)^2$

$4BL^2=4AB^2+AC^2$----(i)

In ΔACM

$4CM^2=4AC^2+AB^2$----(ii)

On adding

$4BL^2+4CM^2=4AB^2+AC^2+4AC^2+AB^2$

$=5AB^2+5AC^2$

$=5(AC^2+AB^2)$

$=5BC^2$

Ie $4BL^2+4CM^2=5BC^2$

(6)AC^2=122=144-----(i)

$AB^2+BC^2= (6\sqrt{3})^2+6^2$

=108+36

AB^2+BC^2= 144---------(ii)

From (i)&(ii)

$AC^2=AB^2+BC^2$(converse of Pythagoras theorem)

$\angle B=90^0$

(7) In ΔPMR

$PR^2= PM^2+ MR^2$

$= 6^2+8^2$

= 36+64

= 100
PR= 10cm
In ΔPQR $PQ^2= QR^2-PR^2$
$= 26^2- 10^2$
$=676-100$
$=576$
PQ= 24cm
Now Area of ΔPQR= ½ x PR x PQ
= ½ x 10 x24
= 120 cm^2

8. Ratio of areas of two similar Δ^s is equal to the ratio of squares of corresponding sides
So Ratio of areas of two similar $\Delta^s = (2x/3x)^2 = 4/9$
So Ratio of areas of two similar $\Delta^s =$ ratio of squares of their corresponding altitudes= 4/9
So, Ratio of corresponding altitudes = 4/9

9. P divide CA in the ratio 2 :1 Therefore
CP = 2/3 AC ……………………………….(i)
QC = 2/3BC ……………………………….(ii)
In Right Triangle ACQ,
$AQ^2 = QC^2 + AC^2$
Or, $AQ^2 = 4/9\ BC^2 + AC^2$ (QC = 2/3 BC)
Or, $9\ AQ^2 = 4\ BC^2 + 9\ AC^2$ ……………..(iii)
Similarly, In Right Triangle BCP
$9BP^2= 9BC^2 + 4\ AC^2$ ……………………..(iv)
Adding eq. (iii) & (iv)
$9(AQ^2 + BQ^2) = 13(BC^2 + AC^2)$
$9(AQ^2 + BQ^2) =13AB^2$

10.In triangle ABD,
PQ !! AB
PQ/AB= DQ/BD
Or, Z/X=DQ/BD…………………………….(i)
In triangle BCD,
PQ !! CD
PQ/CD=BQ/BD
Or, Z/Y=BQ/BD…………………………..(ii)
Adding eq. (i) & (ii)
Z/X+ Z/Y = DQ/BD + BQ /BD = DQ + BQ/BD
Or, Z/X + Z/Y = BD/BD=1
Or, 1/X + 1/y = 1/Z

11. State and Prove Pythagoras Theorem
AP = AB – PB = (18- 12)m = 6m
[PB = CD = pm]
Pc = BD = 8m
In ΔACP
$AC = \sqrt{AP^2 + PC^2}$
$= \sqrt{(8)^2+ (6)^2}$
$= \sqrt{64 + 36} = \sqrt{100} = 10$
AC = 10 m

(12)DE//GF &AC cuts them

⇨$\angle DAG = \angle FGC$(corres. $\angle^s$)

$\angle GDE = 90^0$⇨$\angle GDA = 90^0$

$\Delta ADG \sim \Delta GCF$ (By AA corollary ,shown above)

(ii) similarly$\Delta FEB \sim \Delta GCF$

Since ΔADG &ΔFEB are both similar to ΔGCF

⇨$\Delta ADG \sim \Delta FEB$

(iii)$\Delta ADG \sim \Delta FEB$

AD/FE=DG/FB

⇨AD/DG= EF/EB

(iv) $\Delta ADG \sim \Delta FEB$

AD/FE=DG/FB

⇨AD/DE= DE/EB(FE=DG=DE)

DE^2=AD X EB

(13)(i)distance from the initial point=$\sqrt{3^2+4^2}$

$= \sqrt{25}$

=5m

(ii) Pythagoras theorem

(iii) To save time &energy