MATRICES

KEY POINTS TO REMEMBER

- **Matrix:** A matrix is an ordered rectangular array of numbers or functions . The numbers or functions are called elements of the matrix.
- **Order of a matrix :**A matrix having 'm' rows & 'n' columns is called matrix of order m x n.
- **Zero Matrix:** A matrix having all the elements zero is called zero matrix or null matrix.
- **Diagonal Matrix:** A square matrix is called a diagonal matrix if all it's non diagonal elements are zero.
- **Scalar Matrix:** A diagonal matrix in which all diagonal elements are equal is called a scalar matrix.
- **Identity Matrix:** A scalar matrix in which each diagonal element is 1,is called an identity matrix or a unit matrix. It is denoted by I.

  $I = [a_{ij}]_{n \times n}$ where $a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

- **Transpose of a Matrix:** If $A = [a_{ij}]_{mxn}$ be an m x n matrix then the matrix obtained by interchanging the rows and columns of A is called the transpose of the matrix. Transpose of A is denoted by A' or A^T.

  Properties of the transpose of a matrix.
  1) (A')' = A
  2) (A + B)' = A' + B'
  3) (kA)' = kA' , k is a scalar.
  4) (AB)' = B' A'
- **Symmetric Matrix:** A square matrix $A = [a_{ij}]$ is symmetric if $a_{ij} = a_{ji}\ \forall$ i, j. Also a square matrix A is symmetric if A' = A.
- **Skew Symmetric Matrix:** A square matrix $A = [a_{ij}]$ is skew symmetric if $a_{ij} = - a_{ji}\ \forall$ i, j. Also a square matrix A is symmetric if A' =- A.

ASSIGNMENT

1. If A = diag(1,-1,2) and B = diag(2,3,-1) find A + B, 3A+ 4B.

2. Find a matrix X such that 2A+B+X=0 where A= $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ B = $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$

3. Solve the matrix equation $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$

4. If A = $\begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$ B= $\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ find a matrix C such that 5A+3B+2C is a null matrix.

5. In a certain city there are 30 colleges. Each college has 15 peons , 6 clerks,1typist and 1 section officer. Express given information as a column matrix. Using scalar multiplication find the total no. of posts of each kind in all the colleges.

.6. If A, B, C are three matrices such that A= $\begin{bmatrix} x & y & z \end{bmatrix}$

B = $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ C = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ find ABC

7. If A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ B = $\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$ Is $(A+B)^2 = A^2+2AB+B^2$

8. If A = $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that f(A) = 0. Use this result to find A^5.

9. If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ prove that

i) $A_\alpha . A_\beta = A_{\alpha+\beta}$

ii) $(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ \sin n\alpha & \cos n\alpha \end{bmatrix}$

10. If A and B are square matrices of order n then prove that A and B will commute iff

(A - λi) and (B - λi) commute for every scalar λ.

11. Give an example of 3 matrices A,B,C such that AB = AC but B ≠ C.

12. A matrix X has (a+b) rows and (a+2) columns while the matrix Y has (b+1) rows and

(a+3) columns. Both matrices XY and YX exists. Find a and b. Can you say XY and YX are of the same type ? Are they equal ?

13. Show that the matrix A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ is a root of the equation $A^2 - 4A + I = 0$

14. If A = $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ Prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$

for every positive integer n.

15. If A = diag (a,b,c),show that A^n = diag (a^n , b^n, c^n), for all positive integers n .

16. Give examples of matrices

i) A and B such that $AB \neq BA$.

ii) A and B such that $AB = 0$, but $A \neq 0$, $B \neq 0$.

iii) A and B such that $AB = 0$, but $BA \neq 0$.

iv) A, B and C such that $AB = AC$ but $B \neq C, A \neq 0$

17. If A and B are square matrices of the same order, explain why in general

i) $(A + B)^2 \neq A^2 + 2AB + B^2$

ii) $(A - B)^2 \neq A^2 - 2AB + B^2$

iii) $(A + B)(A - B) \neq A^2 - B^2$

18. Three shopkeepers A,B and C go to a store to buy stationary. 'A' purchases 12 dozen notebooks , 5 dozen pens and 6 dozen pencils . 'B' purchases 10 dozen notebooks , 6 dozen pens and 7 dozen pencils .'C' purchases 11 dozen notebooks , 13 dozen pens and 8 dozen pencils . A notebook costs 40 paise , a pen costs Rs. 1.25 and a pencil costs 35 paise . Use matrix multiplication to calculate each shopkeeper`s bill.

19. Let A be a square matrix , then prove that (AA^T) and (A^TA) are symmetric matrices .

20. Show that all positive integral powers of a symmetric matrix are symmetric .

21. Out of the given matrices choose that matrix which is a scalar matrix

a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

22. If the matrix $A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$ is a symmetric matrix , find x, y , z and t .

23. If $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $AB = I$, find $x+y$.

ANSWERS:

3.x=1,2 y = 1± $\sqrt{10}$ 4. $\begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$ 5. Peons = 450, Clerks = 180 , Typist = 30 , Section officers = 30 6. $ax^2 + 2hxy + by^2 + cz^2 + 2fyz + 2gzx$ 18. Rs. 157.80, Rs. 167.40, Rs. 281.40