

CLASS XII

APPLICATION OF DERIVATIVES

1. Sand is pouring from a pipe at the rate of $12\text{cm}^3/\text{sec}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand-cone increasing when the height is 4cm ?

2. Water is dripping out from a conical funnel at a uniform rate of $4\text{cm}^3/\text{sec}$ through a tiny hole at the vertex in the bottom. When the slant height of the water is 3cm , find the rate of decrease of the slant height of the water cone. Given that the vertical angle of the funnel is 120° .

3. Find the points on the curve $y = x^3 - 11x + 5$ at which the tangent has the equation $y = x - 1$

4. Find the equations of the tangent and normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point, where it cuts x-axis.

5. Find the points on the curve $9y^2 = x^3$ where the normal to curve makes equal intercepts with the axes.

6. Using differentials, find the approximate value of the following up to 3 places of decimals.

a) $3.968^{3/2}$ b) $\frac{1}{\sqrt{25.1}}$

7. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

8. If the radius of a sphere is measured as 9 m with an error of 0.03 m , then find the approximate error in calculating its surface area.

9. Find the intervals in which the functions given below are strictly decreasing or strictly increasing:-

a) $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

b) $f(x) = x^4 - \frac{x^3}{3}$

10. Find the Intervals in which the function f given by

$f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$, is increasing or decreasing.

11. An open box with a square base is to be made out of a given quantity of metal sheet of area c . Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$.

12. Manufacturer can sell x items at a price of rupees $(5 - \frac{x}{100})$ each. The cost price of x items is Rs $(\frac{x}{5} + 500)$. Find the number of items he should sell to earn maximum profit.

13. A point on the hypotenuse of a right angled triangle is at distance a and b from the sides. Show that the length of the hypotenuse is at least $(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{3/2}$.
14. The length of the sides of an isosceles triangle are $9+x^2$, $9+x^2$ and $18-2x^2$ units. Calculate the value of x which makes the area maximum. Also find the maximum area of the triangle.
15. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, find the dimensions of the rectangle *that* will produce the largest area of the window.
16. An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier placed at $(3, 7)$ wants to shoot down the helicopter when it is nearest to him.
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