LIMITS & DERIVATIVES

Class XI

Q.1) $f(x) = \sqrt{2x+3}$ Using first principle method.

Sol.1)

$$f'(x) = \lim_{h\to 0}\left[\frac{\sqrt{2x+2h+3}-\sqrt{2x+3}}{h}\right]$$

Rationalize

$$= \lim_{h\to 0}\left[\frac{\sqrt{2x+2h+3}-\sqrt{2x+3}}{h} \times \frac{\sqrt{2x+2h+3}+\sqrt{2x+3}}{\sqrt{2x+2h+3}+\sqrt{2x+3}}\right]$$

$$= \lim_{h\to 0}\left[\frac{(2x+2h+3)-(2x+3)}{h\left(\sqrt{2x+2h+3}+\sqrt{2x+3}\right)}\right]$$

$$= \lim_{h\to 0}\left[\frac{(2x^2+h^2+2hx+1)(x-3)-(2x^2+1)(x+h-3)}{h(x+h-3)(x-3)}\right]$$

$$= \lim_{h\to 0}\left[\frac{2h}{h\left(\sqrt{2x+2h+3}+\sqrt{2x+3}\right)}\right]$$

$$= \lim_{h\to 0}\left[\frac{2}{\sqrt{2x+3}+\sqrt{2x+3}}\right]$$

$$= \lim_{h\to 0}\left[\frac{2}{2\sqrt{2x+3}}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{2x+3}} \quad \text{ans.}$$

Q.2) $f(x) = x^2 \sin x$ Using first principle method.

Sol.2)

$$f'(x) = \lim_{h\to 0}\left[\frac{(x+h)^2.\sin(x+h) - x^2\sin x}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{(x^2+h^2+2hx)\sin(x+h)-x^2\sin x}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{x^2.\sin(x+h)+h^2\sin(x+h)+2hx.\sin(x+h)-x^2\sin x}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{x^2\{\sin(x+h)-\sin x\}+(h^2+2hx)\sin(x+h)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{x^2.2\cos\left(\frac{2x+h}{2}\right).\sin\left(\frac{h}{2}\right)}{h} + \frac{h(h+2x)\sin(x+h)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{2x^2\cos\left(\frac{2x+h}{2}\right).\sin\left(\frac{h}{2}\right)}{2\times\frac{h}{2}} + (h+2x).\sin(x+h)\right]$$

$$= \lim_{h\to 0}\left(\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\right) \times \lim_{h\to 0}\left(x^2\cos\left(\frac{2x+h}{2}\right)\right) + \lim_{h\to 0}\left((h+2x).\sin(x+h)\right)$$

$$= 1 \times x^2\cos(x) + (2x)\sin x$$

$$\therefore f'(x) = x^2\cos(x) + (2x)\sin x \quad \text{ans.}$$

Q.3)	Differentiate $w.r.t\ x$ (product rule) $f(x) = (ax+b)^m(cx+d)^n$
Sol.3)	We have $f(x) = (ax+b)^m(cx+d)^n$ Differentiate both sides $w.r.t\ x$ (product rule) $f'(x) = (ax+b)^m.\frac{d}{dx}(cx+d)^n + (cx+d)^n.\frac{d}{dx}(ax+b)^m$ $= (ax+b)^m.n(cx+d)^{n-1}.\frac{d}{dx}(cx+d) + (cx+d)^n.m(ax+b)^{m-1}.\frac{d}{dx}(ax+b)$ $= (ax+b)^m.n(cx+d)^{n-1}.(c) + (cx+d)^n.m(ax+b)^{m-1}.(a)$ $= (ax+b)^{m-1}.(cx+d)^{n-1}[(ax+b)nc + (cx+d)ma]$ $= (ax+b)^{m-1}.(cx+d)^{n-1}[ancx + bnc + cmax + dma]$ $f'(x) = (ax+b)^{m-1}.(cx+d)^{n-1}[ax(nc+mc) + (bnc+dma)]$ ans.
Q.4)	Differentiate $w.r.t\ x$ (product rule) $f(x) = (x+\sec x)(x-\tan x)$
Sol.4)	We have $f(x) = (x+\sec x)(x-\tan x)$ Differentiate both sides $w.r.t\ x$ (product rule) $f'(x) = (x+\sec x).\frac{d}{dx}(x-\tan x) + (x-\tan x)\frac{d}{dx}(x+\sec x)$ $= (x+\sec x)(1-\sec^2 x) + (x-\tan x)(1+\sec x\tan x)$ ans.
Q.5)	Differentiate $w.r.t\ x$ (product rule) $f(x) = (x\sin x+\cos x) + (x\cos x-\sin x)$
Sol.5)	We have $f(x) = (x\sin x+\cos x) + (x\cos x-\sin x)$ Differentiate both sides $w.r.t\ x$ (product rule) $f'(x) = (x\sin x+\cos x).\frac{d}{dx}(x\cos x-\sin x) + \frac{d}{dx}(x\cos x-\sin x).\frac{d}{dx}(x\sin x+\cos x)$ $= (x\sin x+\cos x)\left[x.\frac{d}{dx}(\cos x) + \cos x.\frac{d}{dx}(x) - \frac{d}{dx}(\sin x)\right] + [x\cos x-\sin x].\left[x.\frac{d}{dx}(\sin x) + \sin x.\frac{d}{dx}(x) + \frac{d}{dx}(\cos x)\right]$ $= (x\sin x+\cos x)(-x\sin x+\cos x.-\cos x) + (x\cos x-\sin x)(x\cos x+\sin x-\sin x)$ $= (x\sin x+\cos x)(-x\sin x) + (x\cos x-\sin x)(x\cos x)$ $= -x^2\sin^2 x - x\sin x.\cos x + x^2\cos^2 x - x\ sin\ x.\cos x$ $= x^2(\cos^2 x-\sin^2 x) - 2x\sin x\cos x$ $f'(x) = x^2.\cos(2x) - x\sin(2x)$ ans.
Q.6)	Differentiate $w.r.t\ x$ (product rule) $f(x) = x^{-4}(3-4x^{-5})$
Sol.6)	Differentiate $w.r.t\ x$ (product rule)

	$f'(x) = x^{-4}.\frac{d}{dx}\left(3-4x^{-5}\right) + \left(3-4x^{-5}\right).\frac{d}{dx}(x^{-4})$ $= x^{-4}(0+20x^{-6}) + \left(3-4x^{-5}\right)\left(-4x^{-5}\right)$ $= x^{-4}(20x^{-6}) - \left(3-4x^{-5}\right)\left(4x^{-5}\right)$ $= 20x^{-10} - 12x^{-5} + 16x^{-10}$ $= 36x^{-10} - 10x^{-5}$ ans.
Q.7)	Differentiate $w.r.t\ x$ (product rule) $f(x) = \frac{1-\tan x}{1+\tan x}$
Sol.7)	We have $f(x) = \frac{1-\tan x}{1+\tan x}$ $f'(x) = \frac{1-\tan x}{1+\tan x}$ $= \frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}$ $= \frac{\cos x - \sin x}{\cos x + \sin x}$ Differentiate $w.r.t\ x$ (quotient rule) $f'(x) = \frac{(\cos x+\sin x).\frac{d}{dx}(\cos x - \sin x) - (\cos x - \sin x)\frac{d}{dx}(\cos x + \sin x)}{(\cos x + \sin x)^2}$ $= \frac{(\cos x+\sin x)(-\sin x - \cos x) - (\cos x - \sin x)(-\sin x + \cos x)}{(\cos x+\sin x)^2}$ $= \frac{-(\cos x+\sin x)(\cos x+\sin x) - (\cos x - \sin x)(\cos x - \sin x)}{(\cos x+\sin x)^2}$ $= \frac{-(\cos^2 x+\sin^2 x + 2\sin x\cos x) - (\cos^2 x - \sin^2 x - 2\sin x \cos x)}{(\cos x+\sin x)^2}$ $= \frac{-(1+\sin(2x)) - (1-2\sin(2x))}{(\cos x+\sin x)^2}$ $f'(x) = \frac{-2}{(\cos x+\sin x)^2}$ ans.
Q.8)	Differentiate $w.r.t\ x$ (product rule) $f(x) = \frac{x}{\sin^n x}$
Sol.8)	We have $f(x) = \frac{x}{\sin^n x}$ Differentiate $w.r.t\ x$ (quotient rule) $\frac{dy}{dx} = \frac{\sin^n x.\frac{d}{dx}(x) - x.\frac{d}{dx}(\sin^n x)}{(\sin^n x)^2}$

	$= \frac{\sin^n x.(1) - x.n.\sin^{n-1} x.\frac{d}{dx}(\sin x)}{(\sin^{2n} x)^2}$ $= \frac{\sin^n x - nx.\sin^{n-1} x.\cos x}{\sin^{2n} x}$ $= \frac{\sin^{n-1} x(\sin x - nx\cos x)}{\sin^{2n} x}$ $= \frac{\sin x - nx\cos x}{\sin^{2n-n+1}(x)}$ $f'(x) = \frac{\sin x - nx\cos x}{\sin^{n+1} x}$ ans.
Q.9)	$f(x) = \frac{x^2\cos\left(\frac{\pi}{4}\right)}{\sin x}$
Sol.9)	We have $\frac{x^2\cos\left(\frac{\pi}{4}\right)}{\sin x}$ $= f(x) = \frac{1}{\sqrt{2}}.\frac{x^2}{\sin x}$ Differentiate $w.r.t\ x$ (quotient rule) $f'(x) = \frac{1}{\sqrt{2}}\left[\frac{\sin x.\frac{d}{dx}(x^2) - (x^2).\frac{d}{dx}(\sin x)}{(\sin x)^2}\right]$ $= \frac{1}{\sqrt{2}}\left[\frac{\sin x.(2x) - x^2.\cos x}{\sin^2 x}\right]$ $f'(x) = \frac{1}{\sqrt{2}}\left[\frac{2x.\sin x - x^2.\cos x}{\sin^2 x}\right]$ ans.
Q.10)	$f(x) = \frac{\sin x - x\cos x}{x\sin x + \cos x}$
Sol.10)	We have $\frac{\sin x - x\cos x}{x\sin x + \cos x}$ Differentiate $w.r.t\ x$ (quotient rule) $f'(x) = \frac{(x\sin x + \cos x).\frac{d}{dx}(\sin x - x\cos x) - (\sin x - x\cos x).\frac{d}{dx}(x\sin x + \cos x)}{(x\sin x + \cos x)^2}$ $= \frac{(x\sin x + \cos x).\left[\frac{d}{dx}(\sin x) - \left(x.\frac{d}{dx}(\cos x)\right) + \cos x\frac{d}{dx}(x)\right] - (\sin x - x\cos x)\left[x.\frac{d}{dx}(\sin x) + .\frac{d}{dx}(x) + \frac{d}{dx}\cos x\right]}{(x\sin x + \cos x)^2}$ $= \frac{(x\sin x + \cos x).[\cos x - (-x\sin x + \cos x)] - (\sin x - x\cos x)(x.\cos x + \sin x - \sin x)}{(x\sin x + \cos x)^2}$ $= \frac{(x\sin x + \cos x).(x\sin x) - (\sin x - x\cos x)(x\cos x)}{(x\sin x + \cos x)^2}$ $= \frac{x^2\sin^2 x + x\sin x\cos x - x\sin x\cos x + x^2\cos^2 x}{(x\sin x + \cos x)^2}$ $= \frac{x^2(\sin^2 x + \cos^2 x)}{(x\sin x + \cos x)^2}$

	$f'(x) = \frac{x^2}{(x\sin x + \cos x)^2}$ ans.