Q.11) Solve, $\tan x \tan\left(\theta+\frac{\pi}{3}\right)+\tan\left(\theta+\frac{2\pi}{3}\right)=3$

Sol.11) We have, $\tan x \tan\left(\theta+\frac{\pi}{3}\right)+\tan\left(\theta+\frac{2\pi}{3}\right)=3$

$\Rightarrow \tan\theta+\dfrac{\tan\theta+\tan(60)}{1-\tan\theta.\tan(60)}+\dfrac{\tan\theta+\tan(120)}{1-\tan\theta.\tan(120)}=3$

$\Rightarrow \tan\theta+\dfrac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta}+\dfrac{\tan\theta-\sqrt{3}}{1-\sqrt{3}\tan\theta}=3$

$\Rightarrow \tan\theta+\frac{8\tan\theta}{1-3\tan^3\theta}=3$ ………………… (after taking L.C.M)

$\Rightarrow \dfrac{\tan\theta-3\tan^3\theta+8\tan\theta}{1-3\tan^2\theta}=3$

$\Rightarrow \frac{9\tan\theta-3\tan^3\theta}{1-3\tan^2\theta}=3$

$\Rightarrow \frac{3(3\tan\theta-\tan^3\theta)}{1-3\tan^2\theta}=3$

$\Rightarrow 3\tan(3\theta)=3$

$\Rightarrow \tan(3\theta)=1$

$\Rightarrow \tan(3\theta)=\tan\left(\frac{\pi}{4}\right)$

$\Rightarrow 3\theta=n\pi+\frac{\pi}{4}$

$\Rightarrow \theta=\frac{n\pi}{3}+\frac{\pi}{12};\ n\in z$ ans.

Q.12) Solve, $2\tan^2 x+\sec^2 x=2;\ 0\le x\le 2\pi$

Sol.12) We have, $2\tan^2 x+\sec^2 x=2;\ 0\le x\le 2\pi$

$\Rightarrow 2\tan^2 x+1+\tan^2 x=2$

$\Rightarrow 3\tan^2 x=1$

$\Rightarrow \tan^2 x=\frac{1}{3}$

$\Rightarrow \tan x=\pm\frac{1}{\sqrt{3}}$

$\Rightarrow \tan x=\frac{1}{\sqrt{3}}$ and $\tan x=-\frac{1}{\sqrt{3}}$

$x=\frac{\pi}{6}$ or $x=\pi+\frac{\pi}{6}$ $x=\frac{\pi}{6}$ or $x=\frac{7\pi}{6}$	$x=\pi-\frac{\pi}{6}$ or $x=2\pi+\frac{\pi}{6}$ $x=\frac{5\pi}{6}$ or $x=\frac{11\pi}{6}$

$x=\frac{\pi}{6}, x=\frac{7\pi}{6}, x=\frac{5\pi}{6}, x=\frac{11\pi}{6}$ are the possible integers of the given equation where $0\le x\le 2\pi$ ans.

Q.13) If $\cot\theta+\tan\theta=2\operatorname{cosec}\theta$, solve for θ

Sol.13) We have, $\cot\theta+\tan\theta=2\operatorname{cosec}\theta$

$\Rightarrow \frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}=\frac{2}{\sin\theta}$

$\Rightarrow \frac{\cos^2\theta+\sin^2\theta}{\sin\theta.\cos\theta}=\frac{2}{\sin\theta}$

$\Rightarrow \frac{1}{\sin\theta.\cos\theta}=\frac{2}{\sin\theta}$

$\Rightarrow \sin\theta=2\sin\theta.\cos\theta$

$\Rightarrow \sin\theta-2\sin\theta.\cos\theta=0$

$\Rightarrow \sin\theta(1-2\cos\theta)=0$

$\Rightarrow \sin\theta=0$ $\Rightarrow \theta=n\pi$	$\Rightarrow 1-2\cos\theta=0$

	$\Rightarrow \cos\theta = \frac{1}{2}$ $\Rightarrow \cos\theta = \cos\frac{\pi}{3}$ Here, $\alpha = \frac{\pi}{3}$ $\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$ $\therefore \theta = n\pi$ and $\theta = 2n\pi \pm \frac{\pi}{3}; n \in z$ ans.
Q.14)	In any ΔABC show that $a\cos\left(\frac{B-C}{2}\right) = (b+c)\sin\frac{A}{2}$
Sol.14)	R.H.S. $(b+c)\sin\frac{A}{2}$ Using since law $b = k\sin\beta$ and $c = k\sin C$ = $k(\sin B + \sin C).\sin\frac{A}{2}$ = $k.2\sin\left(\frac{B+C}{2}\right).cos\left(\frac{B-C}{2}\right).\sin\frac{A}{2}$………… $\{\sin A + \sin B\ \ formula\}$ = $k.2\sin\left(\frac{B+C}{2}\right).cos\left(\frac{B-C}{2}\right).\sin\frac{A}{2}$ ………. $\{A+B+C=r\}$ = $k.2\cos\left(\frac{A}{2}\right).cos\left(\frac{B-C}{2}\right).\sin\frac{A}{2}$ ………. $\left\{\frac{\pi}{3} \rightarrow change\right\}$ = $k.\left(2\sin\frac{A}{2}.\cos\frac{A}{2}\right).cos\left(\frac{B-C}{2}\right)$ = $k.\sin A.cos\left(\frac{B-C}{2}\right)$ ………. $\{2\sin\theta.\cos\theta = \sin(2\theta)\}$ = $a\cos\left(\frac{B-C}{2}\right)$ = R.H.S. (proved)…………… $\{since\ law\ a = k\sin A\}$
Q.15)	In any ΔABC show that $\frac{\sin(B-C)}{\sin(B+C)} = \frac{b^2-c^2}{a^2}$
Sol.15)	R.H.S. $\frac{b^2-c^2}{a^2}$ Using since law $a = k\sin A, b = k\sin B$ and $c = k\sin C$ = $\frac{k^2\sin^2 B - k^2\sin^2 C}{k^2\sin^2 A}$ = $\frac{\sin^2 B - \sin^2 C}{\sin^2 A}$ = $\frac{\sin(B+C).\sin(B-C)}{\sin^2 A}$ ………. $\{\sin(A+B)\sin(A-B) = \sin^2 A.\sin^2 B\}$ = $\frac{\sin(\pi-A).\sin(B-C)}{\sin^2 A}$ ……………. $\{A+B+C=\pi\}$ = $\frac{\sin A.\sin(B-C)}{\sin^2 A}$ = $\frac{\sin(B-C)}{\sin^2 A}$ = $\frac{\sin(B-C)}{\sin(\pi-(B+C))}$ = $\frac{\sin(B-C)}{\sin(B+C)}$ L.H.S. (proved)
Q.16)	In any ΔABC show that $a^3\sin(B-C) + b^3\sin(C-A) + c^3\sin(A-B) = 0$
Sol.16)	L.H.S. $a^3\sin(B-C) + b^3\sin(C-A) + c^3\sin(A-B)$ Using since law $a = k\sin A, b = k\sin B$ and $c = k\sin C$ = $k^3[\sin^3 A.\sin(B-C) + \sin^3 B\sin(C-A) + \sin^3 C\sin(A-B)]$ = $k^3[\sin^2 A.\sin A.\sin(B-C) + \sin^2 B.\sin B.\sin(C-A) + \sin^2 C.\sin C.\sin(A-B)]$ = $k^3\big[\sin^2 A.\sin\big(\pi-(B+C)\big).\sin(B-C) + \sin^2 B.\sin\big(\pi-(C+A)\big)\sin(C-A) +$ $\sin^2 C.\sin\big(\pi-(A+B)\big).\sin(A-B)\big]$ = $k^3[\sin^2 A.\sin(B+C).\sin(B-C) + \sin^2 B.\sin(C+A)\sin(C-A) +$ $\sin^2 C.\sin(A+B).\sin(A-B)]$

	$= k^3[\sin^2 A.(\sin^2 B - \sin^2 C) + \sin^2 B(\sin^2 C - \sin^2 A) + \sin^2 C(\sin^2 A - \sin^2 B)]$ $= k^3[\sin^2 A.\sin^2 B - \sin^2 A.\sin^2 C + \sin^2 B.\sin^2 C - \sin^2 A.\sin^2 B + \sin^2 C.\sin^2 A - \sin^2 B.\sin^2 C]$ $= k^3 = 0$ R.H.S. (proved) ans.	