SEQUENCE AND SERIES
Class XI

Q.1) The sum of n terms of two A.P.'S are in the ratio $(3n+8):(7n+15)$. Find the ratio of their 12^{th} terms.

Sol.1)

1^{st} A.P.	2^{nd} A.P.
First term: a	First term: a^1
Difference: d	Difference: d^1
12^{th} term: a_{12}	12^{th} term: ${a^1}_{12}$
Sum: S_n	Sum: ${S^1}_n$

To find: $\frac{a_{12}}{{a^1}_{12}}$ i.e., $\frac{a+11d}{a^1+11d^1}$

Given: $\frac{S_n}{{S^1}_n} = \frac{3n+18}{7n+15}$

$\Rightarrow \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a^1+(n-1)d^1]} = \frac{3n+8}{7n+15}$

$\Rightarrow \frac{[2a+(n-1)d]}{[2a^1+(n-1)d^1]} = \frac{3n+8}{7n+15}$

Put $n = 23$ both the sides

$\Rightarrow \frac{2a+22d}{2a^1+22d^1} = \frac{69+8}{161+15}$

$\Rightarrow \frac{2(a+11d)}{2(a^1+11d^1)} = \frac{77}{176}$

$\Rightarrow \frac{a_{12}}{{a^1}_{12}} = \frac{7}{16}$

Hence, the required ratio is $7:16$ ans.

Q.2) The ratio of the sum of m & n terms of an A.P.'S is $m^2:n^2$ show that the ratio of the m^{th} term and n^{th} terms is $(2m-1):(2n-1)$.

Sol.2) To prove: $\frac{a_m}{a_n} = \frac{2m-1}{2n-1}$

Given: $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

$\Rightarrow \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a^1+(n-1)d^1]} = \frac{m^2}{n^2}$

$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$

$\Rightarrow 2an + (nm-n)d = 2am + (nm-m).d = 0$

$\Rightarrow 2a(n-m) + d(nm-n-nm+m) = 0$

$\Rightarrow 2a(n-m) - d(n-m) = 0$

$\Rightarrow (n-m)[2a-d] = 0$

$\Rightarrow (2a-d) = 0$

$\Rightarrow d = 2a$

Now, $\frac{a_m}{a_n} = \frac{a+(m-1)(2a)}{a+(n-1)(2a)}$

$= \frac{a+(1+2m-2)}{a+(1+2n-2)}$

$\Rightarrow \frac{a_m}{a_n} = \frac{2m-1}{2n-1}$ (proved)

Q.3) If the sum of n terms of an A.P. is $pn+qn^2$. Find the common difference.

Sol.3) We have, $S_n = pn + qn^2$.

Put $n = 1, S_1 = p + q$

$\Rightarrow a_1 = p + q$ ……….. $\{\because\ S_1 = a_1\}$

Put $n = 2, S_2 = 2p + 4q$

	$\Rightarrow a_1 + a_2 = 2p + 4q$ .... $\{\because\ S_1 = a_1 + a_2\}$ $\Rightarrow p + q + a_2 = 2p + 4q$ $\Rightarrow a_2 = p + 3q$ Now, $d = a_2 - a_1$ $= (p + 3q) - (p + q)$ $d = 2q$ ans.	
Q.4)	The interior angles of a polygon are in A.P. The smallest angle is 120° & he common difference is 5°. Find the number of sides of the polygon.	
Sol.4)	Let $n \rightarrow$ no. of sides in the polygon Interior angles form an A.P. with $a = 120^{\circ}, d = 5^{\circ}$, no. of term $= n$ Then, $S_n = \frac{n}{2}[240 + (n-1)5]$ $= \frac{n}{2}[240 + 5n - 5]$ $S_n = \frac{n}{2}[5n + 235]$ .... (i) Also, sum of all interior angles in any polygon with n-sides $= (n-2) \times 180^{\circ}$ .... (ii) Equation (i) & (ii) $\Rightarrow \frac{n}{2}[5n + 235] = (n-2) \times 180^{\circ}$ $\Rightarrow 5n^2 + 235n = (n-2) \times 180^{\circ}$ $\Rightarrow 5n^2 + 235n = 360n - 720$ $\Rightarrow 5n^2 + 125n + 720 = 0$ $\Rightarrow n^2 - 25n + 144 = 0$ $\Rightarrow (n-16)(n-9) = 0$ $\Rightarrow n = 16 \text{ or } n = 9$ When $n = 16$, Then, $a_{16} = a + 15d$ $= 120 + 15(5)$ $= 195 > 180^{\circ}$ (not possible $\because$ interior angle cannot > 180°) When $n = 9$, Then, $a_9 = a + 8d$ $= 120 + 8(5)$ $= 160 < 180^{\circ}$ (possible) $\therefore$ no. of sides in the polygon= 9 ans.	
Q.5)	The sum of the first term p, q, r terms of an A.P. are a, b, c respectively. Show that $\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$	
Sol.5)	Let A →1st term of A.P. D → common difference Then $a_p = a = \frac{p}{2}[2A + (p-1)D]$ (or) $\frac{a}{2} = \frac{1}{2}[2A + (p-1)D]$ $\Rightarrow a_q = b = \frac{q}{2}[2A + (q-1)D]$ (or) $\frac{b}{2} = \frac{1}{2}[2A + (q-1)D]$ And $a_r = c = \frac{r}{2}[2A + (r-1)D]$ (or) $\frac{c}{2} = \frac{1}{2}[2A + (r-1)D]$ Now, taking L.H.S., $\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$ Putting value of $\frac{a}{p}, \frac{b}{q}, \frac{c}{r}$ from the above equations:	

	$= \frac{1}{2}[2A + (p-1)D](q-r) + \frac{1}{2}[2A + (q-1)D](r-p) + \frac{1}{2}[2A + (r-1)D](p-q)$ $= \frac{1}{2}\{2A(q-r) + (p-1)D(q-r) + 2A(r-p) + (q-1)D(r-p) + 2A(p-q)$ $+ (r-1)D(p-q)\}$ $= \frac{1}{2}\{2A(q-r) + (p-1)D(q-r) + 2A(r-p) + (q-1)D(r-p) + 2A(p-q)$ $+(r-1)D(p-q)\}$ $= \frac{1}{2}\{2A[q-r+r-p+p-q] + D[pq-r-q+r+rq-pq-r+p+rp-rq-p$ $+q]$ $= \frac{1}{2}[2A(0) + D(0)]$ $= \frac{1}{2}(0)$ $= 0$ R.H.S. ans.
Q.6)	Insert 3 A.M.'S between 3 and 19.
Sol.6)	Here, $a = 3, b = 19$ & $n = 3$ Lt A.M.'S are $A_1, A_2,$ & A_3 Now, $d = \frac{b-a}{n+1} = \frac{19-3}{3+1} = \frac{16}{4} = 4$ $A_1 = a + d = 3 + 4 = 7$ $A_2 = a + 2d = 3 + 8 = 11$ $A_3 = a + 3d = 3 + 12 = 15$ ∴ required no.s are 7,11,15 ans.
Q.7)	For what value of n, $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M. between & b.
Sol.7)	We have, $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ = A.M. $\Rightarrow \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{a+b}{2}$ $\Rightarrow 2a^{n+1} + 2b^{n+1} = (a+b)(a^n + b^n)$ $\Rightarrow 2a^{n+1} + 2b^{n+1} = a^{n+1} + ab^n + ba^n + b^{n+1}$ $\Rightarrow 2a^{n+1} - a^{n+1} + 2b^{n+1} - b^{n+1} = ab^n + ba^n$ $\Rightarrow a^{n+1} - b^{n+1} = ab^n + ba^n$ $\Rightarrow a^{n+1} - ba^n = ab^n - b^{n+1}$ $\Rightarrow a^n(a-b) = b^n(a-b)$ $\Rightarrow a^n = b^n$ $\Rightarrow \frac{a^n}{b^n} = 1$ $\Rightarrow \left(\frac{a}{b}\right)^n = 1$ $\Rightarrow \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^0$ $\Rightarrow n = 0$ ans.
Q.8)	Between 1 and 31, m numbers are inserted so that resulting sequence is an A.P. if the ratio of 7th & $(m-1)^{th}$ number is 5: 9. Find the value of m.
Sol.8)	We have, $a = 1, b = 31$ & $n = m$ Now $d = \frac{b-a}{n+1}$ $\Rightarrow d = \frac{31-1}{m+1} = \frac{30}{m+1}$ Given, $\frac{A_7}{A_{m-1}} = \frac{5}{9}$

	$\Rightarrow \frac{a+7d}{a+(m-1)d} = \frac{5}{9}$ $\Rightarrow \frac{1+7\left(\frac{30}{m+1}\right)}{a+(m-1)\left(\frac{30}{m+1}\right)} = \frac{5}{9}$ $\Rightarrow \frac{m+1+210}{m+1+30m-30} = \frac{5}{9}$ $\Rightarrow \frac{m+211}{31m-19} = \frac{5}{9}$ $\Rightarrow 9m + 1899 = 155m - 145$ $\Rightarrow 146m = 2044$ $\Rightarrow m = \frac{2044}{146} = 14$ $\Rightarrow m = 14$ ans.
Q.9)	If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P. show that a, b, c are also in A.P.
Sol.9)	We have, $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P. Adding 1 in each term $\Rightarrow a\left(\frac{1}{b}+\frac{1}{c}\right)+1, b\left(\frac{1}{c}+\frac{1}{a}\right)+1, c\left(\frac{1}{a}+\frac{1}{b}\right)+1$ are also in A.P. $\Rightarrow a\left[\frac{1}{b}+\frac{1}{c}+\frac{1}{a}\right], b\left[\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right], c\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right]$ are in A.P. $\Rightarrow 2b\left[\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right] = a\left[\frac{1}{b}+\frac{1}{c}+\frac{1}{a}\right] + c\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right]$ $\Rightarrow 2b\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right] = \left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)(a+c)$ $\Rightarrow 2b = a + c$ a, b, c are in A.P. (proved)
Q.10)	Of the sum of three numbers in A.P. is 24 & their product is 440. Find the numbers.
Sol.10)	Let the numbers are $a-d, a, a+d$ Sum = 24 $\therefore a-d+a+a+a+d = 24$ $\Rightarrow 3a = 24$ $\Rightarrow a = 8$ $\Rightarrow$ Product $= 440$ $\Rightarrow (a-d)(a)(a+d) = 440$ Put $a = 8$ $\Rightarrow (8-d)(8)(8+d) = 440$ $\Rightarrow (8-d)(8+d) = \frac{440}{8} = 55$ $\Rightarrow 64 - d^2 = 55$ $\Rightarrow d^2 = 9$ $\Rightarrow d = 3$ & $d = -3$ For $a = 8$ & $d = 3$ No.s are 11,8,5 $\therefore$ required no.s are 5,8,11 (or) 11,8,5