Downloaded from www.studiestoday.com

COMBINATION

Q.36) 4 cards out of 52 cards are chosen. Find no. of ways in which :

Sol.36)

1. 4 cards are chosen:-

(i) 4 cards out of 52 cards can be chosen in = ${}^{52}C_4$ ways = $\frac{52!}{4!48!} = 270725$ ans.

2. 4 cards out of same suit:

(i) There are 4 suits

Diamond	Club	Heart	Spade
(13)	(13)	(13)	(13)

(ii) No. of ways of selecting, 4 diamonds out of 13 diamond cards = ${}^{13}C_4$

(iii) Similarly, ${}^{13}C_4$ ways for selecting 4 spade, 4 clubs & 4 heart

(iv) ∴ required no. of ways of selection = ${}^{13}C_4+{}^{13}C_4+{}^{13}C_4+{}^{13}C_4$ = 4 x ${}^{13}C_4$ = 2860 ans.

3. 4 cards belong to 4 different suits

(i) We have to select 1 card from each suit

(ii) 1 diamond out of 13 diamonds can be selected in = ${}^{13}C_1$ ways

(iii) Similarly, ${}^{13}C_1$ is the no. of ways of selecting 1 club, 1 heart & 1 spade

(iv) ∴ required no. of selection = 13C1 x 13C1x13C1x13C1 = 13 x 13 x 13 x 13 = $(13)^4$ = 28561 ans.

4. All are face cards

(i) There are 12 face cards (4J, 4Q, 4K)\4 face cards out of 12 face cards can be selected in ${}^{12}C_4$ ways = $\frac{12!}{4!8!} = 495$ ans.

5. Two are red & two are black:

(i) Red cards =26 , black cards = 26

(ii) 2 red cards out of 26 red cards can be selected in = ${}^{26}C_2$ ways

(iii) 2 black cards out of 26 can be selected in = ${}^{26}C_2$ ways

(iv) Required no. of selections = ${}^{26}C_2$ x ${}^{26}C_2$ = $\frac{26!}{2!24!} \text{ x } \frac{26!}{2!24!}$ = 325 x 325 = 105625 ans.

6. 4 cards are of same colour:-

(i) 2 cases: either they all are red & all are black

(ii) 4 red cards out of 26 red cards can be selected in = ${}^{26}C_4$ ways

(iii) 4 black cards out of 26 black cards can be selected in ${}^{26}C_4$ ways

(iv) ∴ required no. of ways of selection $= {}^{26}C_4 + {}^{26}C_4 = \frac{26!}{4!22!} + \frac{26!}{4!22!}$ = 14950 + 14950 = 29900 ans.

Q.37) A group consisting of 4 girls & 7 boys. In how many way 5 members are selected such that the team consists:

Sol.37)

1. No girls

(i) Since no girl are to be selected, the remaining 5 are to selected from 7 boys

(ii) Which can be selected in = ${}^{7}C_5$ ways = ${}^{7}C_5$ = ${}^{7}C_2$ = $\frac{7 \text{x} 6}{2}$ = 21 ans.

2. At least 3 boys

Three cases:

B (7)	G (4)
3	2
4	1
5	0

Case: 1) selecting 3 boys & 2 girls which can be selected in = $7c_3 \text{x } 4c_2$ ways = 35 x 6 = 210

Case: 2) selecting 4 boys & 1 girl which can be selected in $7c_3 \text{x } 4c_2$ ways = 35 x 4 = 180

Case: 3) selecting 5 boys & no girl which can selected in = $7c_5 \text{x } 4c_0$ ways = 21 x 1 = 21

∴ required no. of ways of selection $= 210 + 180 + 21 = 411$ ans.

Downloaded from www.studiestoday.com

StudiesToday.com

3. At most 2 boys:

(7)	(4)
B	G
2	3
1	4

Case:1) selecting 2 boys & 3 girls which can be selected in = $7c_2$x $4c_3$ ways = 21 x 4 = 84

Case: 4) selecting 1 boy 4 girls which can be selected in $7c_1$x $4c_4$ ways = 7 x 1 = 7

∴ required no. of ways of selection = 84 + 7 = 91 ans.

4. At least 1 boy & 1 girl

(7)	(4)
B	G
1	4
2	3
3	2
4	1

Case:1) selecting 1 boy 4 girls which can be selected in = $7c_1$x $4c_4$ ways = 7 x 1 = 7

Case:2) selecting 2 boys 3 girls which can be selected in = $7c_2$x $4c_3$ ways = 21 x 4 = 84

Case:3) selecting 3 boys & 2 girls which can be selected in = $7c_3$x $4c_2$ ways = 35 x 6 = 210

Case:4) selecting 4 boys & 1 girl which can be selected in $7c_4$x $4c_1$ ways = 35 x 4 = 180

∴ required no. of ways of selection = case:1 + case:2 + case:3 + case:4
= 7 + 84 + 210 = 180 = 481 ans.

5. At most 1 girl is chosen

(7)	(4)
B	G
4	1
5	0

Case:1) selecting 4 boys & 1 girl which can be selected in = $4c_1$ x $7c_4$ *ways* = 4 x 35 = 180

Case:2) selecting NO girl & 5 boys which can be selected in = $4c_6$ x $7c_5$ *ways* = 1 x 21 = 21

∴ required no. of ways of selections = 180 + 21 = 201

6. A particular boy & a particular girl is always chosen:-
   (i) Let the particular boy is A girl is B
   (ii) They are selected only in 1 way (as they are always selected)
   (iii) Now we have to select 3 persons from the remaining 11 persons
   (iv) Which can be selected in = $11C_3$ ways = 165 ans.

Q.38) A polygon has n sides. Find the number of diagonals?

Sol.38)
(i) A polygon having n sides has n vertices
(ii) Total number of lines that can be drawn using n vertices (points) = $^nC_2$
(iii) Then $^nC_2$ lines also contain n- sides
(iv) ∴ the number of diagonals $^nC_2$ – n = $\frac{n(n-1)}{2} - n = \frac{n^2 - n - 2n}{2} = \frac{n^2 - 3n}{2}$ ans.

Q.39) A polygon has 44 diagonals. Find the number of sides?

Sol.39)
(i) We know that the total no. of diagonals having n-sides = $\frac{n^2-3n}{2}$ (from q.38)
(ii) Given: no. of diagonals 44

∴ $\frac{n^2-3n}{2} = 44$

Downloaded from www.studiestoday.com

	⇨ $n^2 - 3n - 88 = 0$ ⇨ $(n-11)(n+8) = 0$ ⇨ $n = 11$ ⇨ $n = -8$ (no. of sides can never be –n) ∴ there are 11 sides in the polygon	
Q.40)	There are 10 points in a plane, out of which 4 points are collinear. Find no. of straight lines & no. of triangles?	
Sol.40)	1. Total no. of straight lines using 10 points = $10c_2$ (i) No. of straight line using 4 points = $4c_2$ (ii) But 4 collinear points, when join pair wise gives only 1 straight line (iii) ∴ required no. of straight lines = $10c_2 - 4c_2+1$ = 45-6+1=40 ans. 2. Total no. of triangles using 10 points= $10c_3$ (i) No. of triangles using 4 points = $4c_3$ (ii) But 4 collinear points cannot form a triangle (iii) ∴ required no. of triangles = $10_3 - 4c_3$ = 120-4=116 ans.	
Q.41)	There are 'm' no. of horizontal parallel lines & 'n' no. of vertical parallel lines. How many no. of parallelogram can be formed?	
Sol.41)	(i) To form a parallelogram, we require two horizontal lines & two vertical lines (ii) Now two horizontal lines out of 'm horizontal' lines can be selected in = mc_2 ways (iii) Two vertical lines out of 'n vertical' lines can be selected in = $nc_{2\ ways}$ (iv) ∴ the required no. of triangles = mc_2 x nc_2	
Q.42)	From a class of 25 students, 10 are to be chosen for a party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can they be chosen?	
Sol.42)	There are two cases Case:1) three particular students join the party:- (i) Now we have to select 7 student from the remaining 22 students (ii) Which can be selected in $^{22}C_7$ ways Case:2) three particular students do not join the party:- (i) Now we have to choose 10 students from the remaining 22 students (ii) Which can be selected in = $^{22}C_{10}$ ways ∴ required no. of ways of selection = case:1 + case:2 = $^{22}C_7 + {}^{22}C_{10}$ $= \frac{22!}{7!15!} + \frac{22!}{10!12!} = 817190$ ans.	
Q.43)	A boy has 3 library tickets and 8 books of his interest in the library of these 8 books; he does not want to borrow chemistry part 2, unless chemistry part 1 is also borrowed. In how many ways can he choose the three books?	
Sol.43)	There are 2 cases:- Case:1) when chemistry part 1 is borrowed :- (i) Now, he has to select 2 books out of the remaining 7 books (ii) Which can be selected in = $7c_2$ ways Case:2) when chemistry part 1 is not borrowed:- (i) Then, he does not want to borrow chemistry part 2 (ii) Now, he has to select 3 books out of the remaining 6 books (iii) Which can be selected in $6c_{3\ ways}$ ∴ required no. of ways of selection = case:1 + case:2 $= 7c_2 + 6c_3 = 21 + 20 = 41$ ans.	
Q.44)	A box contains 5 red balls & 5 black balls. In how many ways 6 balls be selected such that:	
Sol.44)	1. There are exactly 2 red balls 2. At least 3 red balls	

Downloaded from www.studiestoday.com

Downloaded from www.studiestoday.com

	3. At least 2 red balls 4. At least 2 balls from each colour 5. No. of black balls & no. of white balls are equal 6. Red balls are in majority	
Q.45)	If $2n_{C_3} : n_{C_3}$ = 11:1, find n?	
Sol.45)	We have, $\frac{2n_{C_3}}{n_{C_3}} = \frac{11}{1}$ ⇨ $\frac{\frac{(2n)!}{3!(2n-3)!}}{\frac{n!}{3!(n-2)!}} = \frac{11}{1}$ ....$\left\{n_{C_3} = \frac{n!}{r!(n-r)!}\right\}$ ⇨ $\frac{(2n)!(n-3)!}{(2n-3)!n!} = 11$ ⇨ $\frac{(2n)(2n-1)(2n-2)(2n-3)!(n-3)!}{(2n-3)!\,n(n-1)(n-2)(n-3)!} = 11$ ⇨ $\frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 11$ ⇨ $\frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 11$ ⇨ $8n - 4 = 11n - 22$ ⇨ 3x = 18 ⇨ n = 6 ans.	
Q.46)	If $2n_{C_3} : n_{C_2}$ = 44:3, find n?	
Sol.46)		n = 6