Chapter: - Principal of Mathematical Induction, Complex Numbers and Quadratic Equations

Q1. Prove the following by using the principal of mathematical induction, where n is Natural number:-

(i) $1+6+6^2+6^3+6^4+6^5+\text{-------------}+6^{n-1}=\frac{6^n-1}{5}$

(ii) 4+8+12+----------------+4n=2n(n+1)

(iii) $a+(a+d)+(a+2d)+\text{---------}+[a+(n-1)d]=\frac{n}{2}[2a+(n-1)d]$

(iv) n (n+1)(2n+1) is divisible by 6

(v) (3x6) + (6x9) + (9x12) +----+3n (3n+3) =3n (n+1)(n+2)

(vi) $2^n>n$

(vii) $(1+x)^n>(1+nx)$,for all $n\geq2$ and $x>-1$

(viii) (5^n-1) is divisible by 4

(ix) $(2x5) + (5x8) + (8x11) +\text{----}+ (3n-1)(3n+2) = n(3n^2+6n+1)$

(x). (x^n-y^n) is divisible by (x-y)

(xi) $\frac{1}{1x2}+\frac{1}{2x3}+\frac{1}{3x4}+---+\frac{1}{n(n+1)}=\frac{n}{n+1}$

(xii) $2(7^n)+3(5^n)-5$ is divisible by 24

(xiii) $1+2+3+4+5+\text{-------------}+n=\frac{n(n+1)}{2}$

(xiv) $(2^{3n}-1)$ is divisible by 7

(xv) $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)----\left(1-\frac{1}{n^2}\right)+=\frac{n+1}{2n}$

(xvi) $(2n+1)<2^n$,for all $n\geq3$,

(xvii) $1+3+5+\text{----------------}+(2n-1)=n^2$

(xviii). $(x^{2n-1}-y^{2n-1})$ is divisible by (x+y)

(xix) 7+77+777 +--------+to n terms $=\frac{7(10^{n+1}-9n-10)}{81}$

(xx) $n^3+(n+1)^3+(n+2)^3$ is divisible by 9

(xxi) $1^2+3^2+5^2+7^2+\text{-----------}+(2n-1)^2=\frac{n(4n^2-1)}{3}$

(xxii) $10^n+3(4^{n+2})+5$ is divisible by 9

(xxiii) $\frac{1}{3\times7}+\frac{1}{7\times11}+\frac{1}{11\times15}+---+\frac{1}{(4n-1)(4n+3)}=\frac{n}{3(4n+3)}$

(xxiv) $(2^{2n}-1)$ is divisible by 3

(xxv) (1x2x3) + (2x3x4) + (3x4x5) +---------+n (n+1) (n+2) = $\frac{n(n+1)(n+2)(n+3)}{4}$

Q2. Find the value of $\frac{i^{592}+i^{590}+i^{598}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$ **Ans.** 1

Q3. If $iz^3+z^2-z+i=0$ then show that. $|z|=1$

Q4. Find the value of $2x^3+2x^2-7x+72$ If $x=\frac{3-5i}{2}$ **Ans.**4

Q5. Find the value of x and y if 4x-i(3x-y)=3+6i. **Ans.** x=3/4, y=33/4.

Q6. Express $(\sqrt{3}-2i)+\sqrt{3}-(-2-7i)$ In the form of x+iy **Ans.** $(2\sqrt{3}+2)+5i$

Q7. Express $3i^3(15i^6)$ In the form of x+iy **Ans.** $(0+45i)$

Q8. Find the multiplicative inverse of $(\sqrt{5}+3i)$ **Ans.** $\frac{\sqrt{5}}{14}-\frac{3}{14}i$

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Q9. If $\frac{a+ib}{c+id}=x+iy$ then show that. $x^2+y^2=\frac{a^2+b^2}{c^2+d^2}$

Q10. If $z=\frac{a+ib}{a-ib}$ then prove that. $|z|=1$

Q11. Find modulus and amplitude(argument) of following complex numbers:-

(i).z=1-cosα+isinα, 0<α<π,(ii)z= 1+cosα+isinα , 0<α<π, **Ans**. (i)2sin$\frac{\alpha}{2},\frac{\pi}{2}-\frac{\alpha}{2}$,(ii)2cos$\frac{\alpha}{2},\frac{\alpha}{2}$,

Q12. Write the following complex numbers in the polar form:-

(i). $z=-3\sqrt{2}+3\sqrt{2}i$ (ii) $z=\frac{1}{1+i}$ (iii) $z=\frac{1+3i}{1-2i}$ (iv) $z=\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$

Ans. (i). $6\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$ (ii) $\frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)$ (iii) $\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$ (iv) $2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

Q13.Find the square –roots of following complex numbers:-

(i). 3+4i (ii)-15 +8i (iii) 5-12i (iv) -15-8i (v)-3-4i (vi)2-2$\sqrt{3}$ i (vii)8+6i (viii)7-24i

Ans. (i). ±(2+i) (ii)±(1+4i) (iii)±(3-2i) (iv)±(1-4i) (v)±(1-2i) (vi)±($\sqrt{3}$ -i) (vii)±(3+i) (viii)±(4-3i).

Q14. Solve the following equations:-

(i) $|z|=z+\overline{(1-2i)}$ (ii) $\text{Re}(z^2)=0, z\bar{z}=4$ (iii) $z^2=\bar{z}$ **Ans.** (i) $\frac{3}{2}-2i$ (ii) $\pm\left(\sqrt{2}\pm\sqrt{2}i\right)$(iii) $-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$

Q15. If z_1, z_2, z_3 are complex numbers such that $|z_1|=|z_2|=|z_3|=\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|=1$ then find the value of

$|z_1+z_2+z_3|$ **Ans**. 1

Q16. Solve the following quadratic equations by factorization method:-

(i).x^2+2x+5= 0 (ii) 4x^2-12x+25=0 **Ans.**(i)x=(-1+2i)and (-1-2i)(ii) $x=\frac{3}{2}+2i$ and $\frac{3}{2}-2i$

Q17. Solve the following quadratic equations:-

(i).2x^2+ $\sqrt{15}$ ix-i= 0 (ii)x^2-x+(1+i) =0 (iii). ix^2-x+12i=0 (iv) $x^2-(3\sqrt{2}-2i)x-\sqrt{2}i=0$

(v) $x^2-(\sqrt{2}+i)x+\sqrt{2}i=0$ (vi) $2x^2-(3+7i)x+(9i-3)=0$ (vii) $x^2-(3\sqrt{2}-2i)x+6\sqrt{2}i=0$

(viii) 2x^2-4x+3=0 (ix)x-2=x^2 (x) 13x^2+7x+1=0 (xi) $x^2+\left(\frac{ax}{x+a}\right)^2=3a^2$

Ans. (i). $x=\frac{1+\left(4-\sqrt{15}\right)i}{4}$ and $\frac{1-\left(4+\sqrt{15}\right)i}{4}$ (ii) x= (1-i)and (i) , (iii) x=(-4i) and(3i)

(iv) $x=\frac{\left(3\sqrt{2}-2i\right)}{2}\pm\frac{\left(4-\sqrt{2}i\right)}{2}$ (v)x= $\sqrt{2}$ and i (vi)x= $\frac{3+i}{2}$ and 3i (vii).x= $3\sqrt{2}$ and 2i (viii) $x=1\pm\frac{i}{\sqrt{2}}$,

(ix) $x=\frac{-1\pm\sqrt{7}i}{-2}$, (x) $x=\frac{-7\pm\sqrt{3}i}{26}$, (xi) $x=\frac{3a\pm i\sqrt{3}a}{2}, \frac{a}{2}\left(1\pm\sqrt{5}\right)$

--**Best of Luck**--