Chapter I

Linear Equations in two variables in two variables

Linear equation in two variables can be written as :-

$ax + by + c = 0,\ a \neq 0,\ b \neq 0$ and a, b and c are real numbers.

This equation has infinitely many solution. When we consider two linear equation together then we say it a system of two variable x and y. such as.

5x + 3y + 2 = 0
2x - 3y + 5 = 0

Algebraic solution of system of linear equation:

1. Elimination by substitution:

Let us consider a system of linar equations:

$2x - 7y = 1$ --------(1)
$4x + 3y = 15$ -------(2)

From equation (1) we get

x = (1 + 7y)/2

Putting the value of x in e.q. (2) we get

$$4\left(\frac{1+7y}{2}\right)+3y=15$$

Or, 2 + 14y + 3y = 15

Or, 17y = 13

Or, y = 13/17

Putting value of y in eq (1) we get

$$2x-7\times\frac{13}{17}=1$$

Or, 2x = 1 + 91/17

Or, 2x = (17+91)/17

Or, x = 108/34

Or, x = 54/17

$\therefore\ x = \frac{54}{17}, y = \frac{13}{17}.$

2. Elimination by equating:

Let us consider a system of equations:

$-6x + 5y = 2$ -------------(1)

$-5x + 6y = 9$ -------------(2)

From equation (1) we get

$y = \frac{2+6x}{5}$ ----------------- (3)

and from eq. (2) we get

$y = \frac{9+5x}{6}$ ----------------- (4)

From (3) and (4) we get

$\therefore\ \frac{2+6x}{5} = \frac{9+5x}{6}$

or, 12+36x = 45+25x

or, 11x = 33

or, x = 3

Putting x = 3 in (3) we get

$y = \frac{2+6\times 3}{5}$

= 20/5

= 4

$\therefore\ x = 3, y = 4$

3. Elimination by equating coefficients:

Let us consider a system of equations:

x - 5y = 11 ----------(1)

2x + 3y = -4 ---------(2)

Multiplying equation (1) by 2 and then subtracting eq. (2) from this we get

$$\begin{array}{r} 2x - 10y = 22 \\ \_\, 2x \pm 3y = \mp 4 \\ \hline -13y = 26 \end{array}$$

Or, y = -2

Patting y = -2 in eq (1) we get

x = -5 (-2) = 11

Or, x + 10 = 11

Or, x = 1

$\therefore\ x = 1, y = -2.$

4. Solution by cross Multiplication:

Let us consider a system of equations:

$$a_1x + b_1y = c_1 = 0, a_1 \neq 0, b_1 \neq 0 \qquad (1)$$
$$a_1x + b_1y = c_1 = 0, a_1 \neq 0, b_1 \neq 0 \qquad (2)$$

From equation (1) we have

$$y = \frac{a_1x + c_1}{-b_1}, (b_1 \neq 0)$$

Putting this value of y in eq (2) we get

$$a_1x + b_1(\frac{a_1x + c_1}{-b_1}) + c_2 = 0$$

Or, $b_1a_2x - b_2a_1x - b_2c_1 + b_1c_2 = 0$

$$(a_1b_2 - a_2b_1)x = b_1c_2 - b_2c_1 - (3)$$

Similarly by eliminating x, we get

$$(a_1b_2 - a_2b_1)y = c_1a_2 - c_2a_1 - (4)$$

Here we see that (3) and (4) are linear equation in one variable only.

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

$$and \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Provided $a_1b_2 - a_2b_1 \neq 0$.

The above results can be written as:

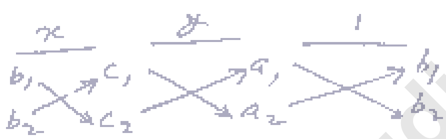
$$\frac{x}{b_1c_2 - b_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

and $\dfrac{y}{c_1a_2 - c_2a_1} = \dfrac{1}{a_1b_2 - a_2b_1}$

and combining these two we get

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

The above result can be written in a picture form as:



When $a_1b_2 - a_2b_1 = 0$, then we can not divide equation's (3) and (4) by $a_1b_2 - a_2b_1$ to fine the value of x and y. if

$a_1b_2 - a_2b_1 = 0$

then $a_1b_2 - a_2b_1$

$$Or, \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \quad as \;\; a_2 \neq 0, b_2 \neq 0$$

$$Let \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{1}{k}, k \neq 0$$

$\therefore\ a_2 = ka_1$ and $b_2 = kb_1$

If $c_2 = kc_1$, then equation (2) reduces to

$ka_1x + kb_1y + kc_1 = 0$

Or, $k(a_1x+b_1y+c_1)=0$

Or, $a_1x+b_1y+c_1=0$

Which is equation (1). Here we see that every solution of equation (1) is a solution of equation (2) hence the system has infinite solutions.

If $c_2 \neq kc_1$, equation on (2) reduces to $ka_1x+kb_1y+c_2=0$

Or, $k(a_1x+b_1y)+c_2=0$

Or, $k(-c_1)+c_2=0$

Or, $C_2 = KC_1$

But this is not true. Therefore, no solution exists.

· The system of equations

$$a_1x+b_1y+c_1=0$$
$$a_2x+b_2y+c_2=0$$

has exactly one or, unique solution if $a_1b_2-a_2b_1 \neq 0$

i. e; if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

has no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

and has infinite solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Example 1. Solve the system of equation:

$$x-4y+14=0$$
$$3x+2y-14=0$$

Solution :

Here , $a_1=1, b_1=-4, c_1=14$
$a_2=3, b_2=2, c_2=-14$

and $a_1b_2 - a_2b_1$

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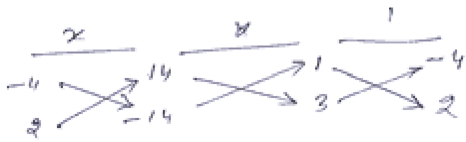
=1 x 2 - 3 x (-4)

= 2 + 12

$= 14 \neq 0$

$\because$ The system has unique solution.



$$\frac{x}{(-4)\times(-14)-2\times 14}=\frac{y}{14\times 3-(-14)\times 1}=\frac{1}{1\times 2-3\times(-4)}$$

Or, $\dfrac{x}{56-28}=\dfrac{y}{42+14}=\dfrac{1}{2+12}$

Or, $\dfrac{x}{28}=\dfrac{y}{56}=\dfrac{1}{14}$

Or, x = 28/14 = 2,

Or, y = 56/14 = 4

$\therefore\ x = 2, y = 4$

Example 2. Find the value of p for which the following system of eqation have exactly one solution:

$$px + 2y = 5$$
$$3x + y = 1$$

Solution: - here we have

$a_1 = p, b_1 = 2, a_2 = 3, b_2 = 1$

$$\begin{aligned} a_1b_2 - a_2b_1 &= p\times 1 - 3\times 2 \\ &= p - 6 \end{aligned}$$

as the system of equation have exactly one solution

$\therefore\ p - 6 \neq 0$

$or,\quad p \neq 6$

Hence, the system of equations have exactly one solution for all value of a except 6.

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Example 3. Solve the following system of equation for x and y

$$\frac{x}{a}+\frac{y}{b}-2=0------------(1)$$
$$ax-by+b^2-a^2=0--------(2)$$

Solution :- Multiplying equation (1) by b^2 and then adding to (2) we get

$$=\frac{b^2x}{a}+\frac{b^2y}{b}-\underline{2b^2}+ax-by\ a^2+\underline{b^2}-\underline{a^2}=0$$

$$Or,\ \frac{b^2x}{a}-ax-a^2-b^2=0$$

$$Or,\ (a^2+b^2)\frac{x}{a}\qquad =a^2+b^2$$

$$Or,\qquad x\quad =a^2+b^2\times\frac{a}{a^2+b^2}$$

Or, x = a

Putting x = a in equation (1) we get

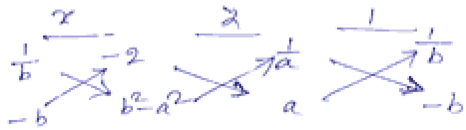
$$\frac{a}{a}+\frac{y}{b}-2=0$$
$$Or,\quad \frac{y}{b}-1=0$$
$$Or,\quad \frac{y}{b}\quad =1$$
$$Or,\quad y\quad =b$$

· x = a, y = b.

Alternative Method:-

$$a_1=\frac{1}{a},\quad a_2=a,\quad b_1=\frac{1}{b},$$
$$b_2=-b,\ c_1=-2\quad c_2=b^2-a^2$$

$$a_1b_2-a_2b_1=\frac{1}{a}\times(-b)-a\times\frac{1}{b}$$
$$=-\frac{a^2+b^2}{ab}$$
$$\neq\ 0$$



$$\frac{x}{\frac{b^2-a^2}{b}-2b}=\frac{y}{-2a-\frac{b^2-a^2}{a}}=\frac{}{-\frac{b}{a}-\frac{a}{b}}$$

$$Or,\ \frac{bx}{b^2-a^2-2b^2}=\frac{ay}{-2a^2-b^2+a^2}=\frac{ab}{-b^2-a^2}$$

$$Or,\ \frac{bx}{-(a^2+b^2)}=\frac{ay}{-(a^2+b^2)}=\frac{ab}{-(a^2+b^2)}$$

$$\therefore\ x=\frac{ab}{-(a^2+b^2)}\times\frac{-(a^2+b^2)}{b}=a$$

$$y=\frac{ab}{-(a^2+b^2)}\times\frac{-(a^2+b^2)}{a}=b$$

$$\therefore\ x=a,\quad y=b$$

Example 4. Solve the following system of equation:

$$\frac{11}{v}-\frac{7}{u}=1----------(1)$$

$$\frac{9}{v}-\frac{4}{u}=6----------(2)$$

Solution: multiplying equation (1) by 4 and equation (2) by 7 we get

$$\frac{44}{v}-\frac{28}{u}=4----------(3)$$

$$\frac{63}{v}-\frac{28}{u}=42--------(4)$$

Subtracting (3) fr0m (4) we get

$$\left[\frac{63}{v}-\frac{28}{u}\right]-\left[\frac{44}{v}-\frac{28}{u}\right]=42-4$$

$$Or,\ \frac{63}{v}-\frac{28}{u}\quad-\ \frac{44}{v}+\frac{28}{u}=38$$

Or, 19/v = 38

Or, 36 v = 19

Or, v = 19/38

Or, v = 1/2

Putting v = 1/2 in (1) we get

$$\frac{11}{1/2} - \frac{7}{u} = 1$$

Or, 22 - 7/u = 1

Or, 21 = 7/u

Or, 21u = 7

Or, u = 7/21

Or, u = 1/3

$$\therefore u = \frac{1}{3}, \quad v = \frac{1}{2}$$

Example 5. Find the solution of following system of equation such that $u \neq 0, v \neq 0$:

$$2u + v = \frac{7}{3}uv$$

$$u + 3v = \frac{11}{3}uv$$

Solution: Dividing both sides of the given equations by uv we get

$$\frac{2}{v} + \frac{1}{u} = \frac{7}{3} - - - - - - - - - (1)$$

$$\frac{1}{v} + \frac{3}{u} = \frac{11}{3} - - - - - - - - (2)$$

Multiplying equation (2) by 2 and then subtracting equation (1) we get

$$\left[\frac{2}{v} + \frac{6}{u}\right] - \left[\frac{2}{v} + \frac{1}{u}\right] = \frac{22}{3} - \frac{7}{3}$$

$$\frac{2}{v} + \frac{6}{u} - \frac{2}{v} + \frac{1}{u} = \frac{15}{3}$$

$$Or, \qquad \frac{5}{u} = \frac{15}{3}$$

Or, 15u = 15

∴ u = 1

Putting u = 1 in equation on (1)

$$\frac{2}{v}+\frac{1}{1}=\frac{7}{3}$$

$$\frac{2}{v}=\frac{7}{3}-1$$

Or, 2/v = (7-3)/3

Or. 2/v = 4/3

Or, 4v = 6

Or, v = 6/4

Or, v = 3/2

$$\therefore u=1, \quad v=\frac{3}{2}$$

Example 6: Solve for x and y:

$$\frac{44}{x+y}+\frac{30}{x-y}=10$$

$$\frac{55}{x+y}+\frac{40}{x-y}=13 \qquad [2002C, OD]$$

Solution: putting x + y =u and x - y = v the given equation reduces to

$$\frac{44}{u}+\frac{30}{v}=10------------(1)$$

$$\frac{55}{u}+\frac{40}{v}=13------------(2)$$

Multiplying equation (1) by 4 and equation (2) by 3 we get

$$\frac{176}{u}+\frac{120}{v}=40------------(3)$$

$$\frac{165}{u}+\frac{120}{v}=39------------(4)$$

Subtracting (4) from (3) we get

11/u = 1

Or, u = 11

Or, $x + y = 11 - - - - - - - - (5)$

Putting u = 11 in (1) we get

$$\frac{44}{11} + \frac{30}{v} = 10$$

$$Or, \quad \frac{30}{v} = 6$$

$$Or, \quad v = 5$$

Or, $x - y = 5 - - - - - - - - - - - - - (6)$

Adding (5) and (6) we get

2x = 16

x = 8

Putting x = 8 in (5) we get

8 + y = 11

y = 3

$\therefore\ x = 8, \quad y = 3$

Example 7. Solve the following system of equation:

$$4x + \frac{6}{y} = 15 \ and \ 6x - \frac{8}{y} = 14 \qquad [1994, OD]$$

Solution:-

$$4x + \frac{6}{y} = 15 - - - - - - - - (1)$$

$$6x - \frac{8}{y} = 14 - - - - - - - - (2)$$

Multiplying equation (1) by 4 and (2) by 3 and then adding we get

$$16x + \frac{24}{y} = 60$$
$$18x - \frac{24}{y} = 42$$
$$34x \quad = 102$$
$$Or, \quad = 3$$

Putting x = 3 in (i) we get

$$4 \times 3 + \frac{6}{y} = 15$$
$$Or, \quad \frac{6}{y} = 3$$
$$Or, \quad y = 2$$
$$\therefore x = 3, \quad y = 2$$

Example 8. Find the value of a and b for which the following system of equations has infinitely many solution; [2002 C, D]

$$(2a-1)x - 3y = 5, \ \ 3x + (b-2)y = 3$$

Solution:-

$$\frac{a_1}{a_2} = \frac{2a-1}{3}, \quad \frac{b_1}{b_2} = \frac{-3}{b-2}, \quad \frac{c_1}{c_2} = \frac{5}{3}$$

For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\frac{2a-1}{3} = \frac{-3}{b-2} = \frac{5}{3}$$

Taking 1st and 3rd ratio

$$\frac{2a-1}{3} = \frac{5}{3}$$
$$2a - 1 = 5$$
$$2a = 5 + 1 = 6$$
$$a = 3$$

Taking last two retio

$$\frac{-3}{b-2} = \frac{5}{3}$$

5b - 10 = -9

5b = -9 + 10 = 1

b = 1/5

$$\therefore a = 3, \quad b = \frac{1}{5}$$

Example 9. Determine the value of k so that the following linear equations have no solution:

$$(3k+1)x + 3y - 2 = 0$$
$$(k^2+1)x + (k-2)y - 5 = 0$$

Solution:-

$$\frac{a_1}{a_2} = \frac{3k+1}{k^2+1}, \frac{b_1}{b_2} = \frac{3}{k-2}, \frac{c_1}{c_2} = \frac{2}{5}$$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{3k+1}{k^2+1} = \frac{3}{k-2} \neq \frac{2}{5}$$

$$Now, \frac{3k+1}{k^2+1} = \frac{3}{k-2}$$

Or, (k - 2) (3k + 2) = 3(k^2 + 1)

Or, 3k^2 - 5k - 2 = 3k^2 + 3

Or, -5k = 5

Or, k = -1

Exercise - 1

1. For what value of k will the following system of linear equations have no solution?

$3x + y = 1, (2k - 1)x + (k - 1)y = 2k + 1$

2. Determine the value of c for which the following system of linear equations has no solution:

$cx + 3y = 3; 12x + cy = 6$

3. For what value of k will the system of equations

$x + 2y = 5; 3x + ky + 15 = 0$

has (i) unique solution (ii) no solution.

4. For what value of k, the following system of equations have (i) a unique solution, (ii) no solutions:

$kx + 2y = 5; 3x - 4y = 10$

5. Find the value of a and b for which the following system of linear equations has infinite numbers of solutions:

$2x - 3y = 7$
$(a + b)x - (a + b - 3)y = 4a + b.)$

Solve the following system of linear equations

6. For the value of k, for which the following system of linear equations has in finite number of solutions:

$2x + 3y = 7$
$(k - 1) x + (k + 2)y = 3k$

solve the following system of linear equations:

7. $3x - 5y = -1, x - y = -1$

8. $x + y = 7, 5x + 12y = 7$

9. $3x - 4y = 1, 4x - 3y = 6$

10. $x + y/2 = 4, x/3 + 2y = 5$

11. $x/3 + y/4 = 4, 5x/6 - y/8 = 4$

12. $2/x + 2/3y = 1/6, 3/x + 2/y = 0$

13. $2u + 15v = 17uv, 5u + 5v = 36uv$

14. $3(a + 3b) = 11ab, 3(2a + b) = 7ab$

15. $2x/a + y/b = 2, x/a - y/b = 4$

16. $x/a - y/b = 0$, $ax + by = a^2 + b^2$

17. $(a - b)x + (a + b)y = a^2 - 2ab - b^2$, $(a + b)(x + y) = a^2 + b^2$

18. $x/a + y/b = a + b$, $x/a^2 + y/b^2 = 2$

19. $ax + by = c$, $bx + ay = 1 + c$

20. $x + y = a - b$, $ax - by = a^2 + b^2$

Answers

(1) 2	**(2)** 6	**(3)** (i) $k \neq 6$ (ii) $k = 6,\ k \neq -6$		
(4) (i) $k \neq -\frac{3}{2}$ (ii) $k = \frac{3}{2},\ k \neq -\frac{3}{2}$		**(5)** a = -5, b = -1	**(6)** 7	**(7)** -2, -1
(8) 11, -4	**(9)** 3, 2	**(10)** 3, 2	**(11)** 6, 8	**(12)** 6, -4
(13) 5, 1/7	**(14)** 1, 3/2	**(15)** 2a, -2b	**(16)** a, b	**(17)** a +b, -2ab/(a + b)

(18) a^2, b^2 **(19)** $(bc - ac + b)/(b^2 - a^2), (bc - ac - a)/(b^2 - a^2)$

Graphical solution of system of linear equations:

Example 10. Solve graphically the system of equations:

$$2x - 3y = 5 - - - - - - - - (1)$$
$$3x + 4y + 1 = 0 - - - - - - -(2)$$

Solution: To draw the graphic of equations (1) and (2) we find three solutions of each of the equations (1) and (2)

$$2x - 3y = 5 - - - - - - -(1)$$

Or, $2x - 5 = 3y$

$$\therefore\ y = \frac{2x-5}{3}$$

$When\ x = -2,\ y = \frac{2(-2)-5}{3} = -3$

$When\ x = 1,\ y = \frac{2(1)-5}{3} = -1$

$When\ x = 4,\ y = \frac{2(4)-5}{3} = 1$

x	1	-2	4
y	-1	-3	1

Now we plot the print (1, -1) (-2, -3) and (4, 1). After joining we get a straight line.

And 3x + 4y + 1 = 0

$$y = -\frac{3x+1}{4}$$

$When\ x = 1, y = -\frac{3\times 1+1}{4} = -1$

$When\ x = -3, y = -\frac{3(-3)+1}{4} = 2$

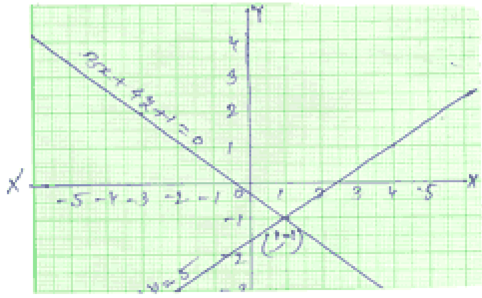
$When\ x = 5, y = -\frac{3(5)+1}{4} = -4$

x	1	-3	5
y	-1	2	-4

Again we plot the point (1, -1), (-3, 2) and ((5, -4) on the same graph paper and join then.

Now, we get see that the lines of these two equations intersect at (1, -1).

$\therefore\ x = 1\ and\ y = -1$. Is the solution of the system of equations.



Example 11. Solve graphically the system of linear equations:

$$4x + 6y = 9$$
$$2x + 3y = -11$$

Solution:- Let us take

$$4x + 6y = 9 ----------(1)$$

$$y = \frac{9 - 4x}{6}$$

When $x = \mathbf{3},\ y = \frac{\mathbf{9 - 4 \times 3}}{\mathbf{6}} = -\frac{\mathbf{1}}{\mathbf{2}}$

When $x = 0,\ y = \frac{9 - 4 \times 0}{6} = \frac{3}{2} = 1.5$

When $x = -3, y = \frac{9 - 4 \times (-3)}{6} = -\frac{7}{2} = 3.5$

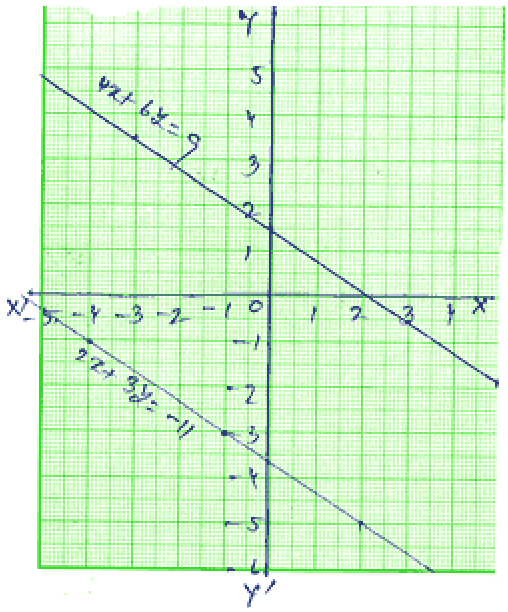
x	0	3	-3
y	1.5	-0.5	3.5

We plot the point (0, 1.5) (3, -0.5) and (-3, 3.5) on a graph paper and join them which is a straight line.

And 2x +3y = -11

$$y = \frac{-(2x + 11)}{3}$$

When $x = -1, y = \frac{-[2 \times (-1) + 11]}{3} = -3$



When $x = 2,\ y = \dfrac{-(2 \times 2 + 11)}{3} = -5$

When $x = -4,\ y = \dfrac{-[2(-4) + 11]}{3} = -1$

x	-1	-4	>2
y	-3	-1	-5

Again we plot the points (-1, -3), (-4, -1) and (2, -5) on the same graph paper and join then we get a straight line which is parallel to previous line. The lines do not intersect and hence we get no solution.

Example 12. Solve the following system of equation graphically.

$$x - 2y = 5 ---------- (1)$$
$$2x - 4y - 10 = 0 ------- (2)$$

Solution:- Let us take the equation

$$x - 2x = 5 ---------- (1)$$
$$y = \frac{x-5}{2}$$

When $x = 1,\ \ y = \dfrac{1-5}{2} = -2$

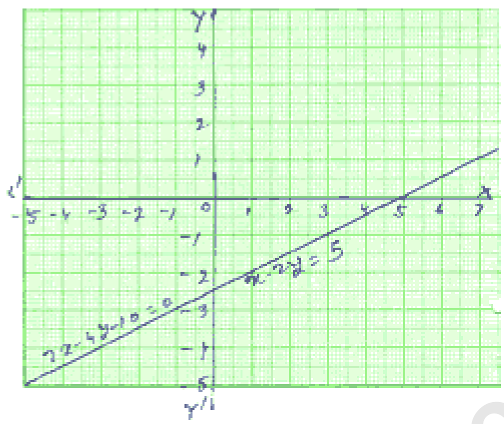
When $x = 3,\ \ y = \dfrac{3-5}{2} = -1$

When $x = 5, \ y = \frac{5-5}{2} = 0$

x	1	3	5
y	-2	-1	0

We plot the points (1, -2), (3, -1) and (5, 0) on a graph paper and join them. We get a straight line.

Now, we take equation $2x - 4y - 10 = 0$



$$y = \frac{2x-10}{x} = \frac{x-5}{2}$$

When x = 1, y = -2

When x = 3, y = -1

When x = 5, y = 0.

We see the points are same as in equation (1). Hence these points line on the previous line. If we consider the joins of these points separately, we can say that one line is totally covered by the other hence: there are infinitely many solution of the system of equation.

Example 13. Draw the graphs of the equations

$$4x - y = 4 - - - - - - - - -(1) \text{ and}$$
$$4x + y = 12 - - - - - - - - -(2)$$

Deter mine the vertices of the triangle formed by the lines representing these equations and the x-axils. Shade the triangular region so formed.

Solution: - Let us take the equation

$$4x - y = 4$$

$\therefore\ y = 4x - 4$

When, $x = 0,\ y = 4 \times 0 - 4 = -4$

When, $x = 1,\ y = 4 \times 1 - 4 = 0$

When, $x = 2,\ y = 4 \times 2 - 4 = 4$

x	o	1	2
y	-4	0	4

We plot the points (0, -4), (1, 0) and (2, 4) on the graph paper and join them. We get a straight line. Now we take the line AB.

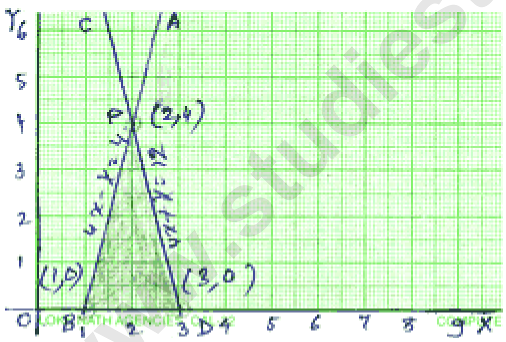
4x + y = 121

$\therefore\ y = 12 - 4x$

When x = 2, y = 12 - 4 x 2 = 4

When x = 3, y = 12 - 4 x 3 = 0

When x = 4, y = 12 - 4 x 4 = -4.



x	2	3	4
y	4	0	-4

We plot the points (2, 4) (3, 0) and (4, -4) on the same graph paper. on joining them we get a line CD which intersect previous line AB. at P (2, 4)

AB intersects the x-axis at (1, 0) and CD intersects the x-axis at (3, 0)

Hence the vertices of the triangle PBD are (2, 4), (1, 0) and (3, 0) . The required region is shaded.

1. Solve the following system of equations graphically:

2. Solve the following system of linear equations graphically and then find the points where the lines meet y-axis:

(i) $2x + y - 5 = 0$
$x + y - 3 = 0$

(ii) $2x - y - 5 = 0$
$x - y - 3 = 0$

(iii) $3x + y - 5 = 0$
$2x - y - 5 = 0$

(iv) $2x + 3y - 12 = 0$
$2x - y - 4 = 0$

(v) $2x + y - 11 = 0$
$x - y - 1 = 0$

(vi) $2x - y = 1$
$x + 2y = 8$

3. Solve the following system of linear equations graphically and shade the area bounded by these lines and y-axis:

(i) $x + 2y - 7 = 0$
$2x - y - 4 = 0$

(ii) $x - y = 1$
$2x + y = 8$

(iii) $3x + y - 11 = 0$
$x - y - 1 = 0$

(iv) $2x - y = 8$
$8x + 3y = 24$

4. Draw the graph of the following equations and solve graphically shade the region bounded by these lines and x-axis. Also calculate the area bounded by these lines and x-axis:

(i) $x - y + 1 = 0$
$3x + 2y - 12 = 0$

(ii) $4x - 3y + 4 = 0$
$4x + 3y - 20 = 0$

(iii) $2x + y = 6$
$2x - y + 2 = 0$

(iv) $2x + 3y = 12$
$x - y = 1$

5. Determine graphically the co-ordinates of the vertices of a triangle, the equation of whose sides are given:

(i) $y = x$; $y = 2x$; $x + y = 6$ (ii) $y = x$; $3y = x$; $x + y = 8$

(iii) $x + y = 5$; $x - y = 5$; $x = 0$

Applications to practical problems:

There are some practical problems which can be sowed by reducing then to linear equations and then sowing then. Let us try to solve some of them;

Example 14. Five years ago, Neeta was trice as old as Reeta. Ten years later, Neeta will be twice as old as Reeta, How old are Neeta and Reeta now?

Solution:- Let present age of Neeta be x years and that of Reeta be y years.

5 years ago age of Neeta = $x - 5$

and 5 years ago age of Reeta = $y - 5$

$$A/Q \quad x - 5 = 3(y - 5)$$
$$Or, \quad x - 3y + 10 = 0 ----------(1)$$

10 years later age of Neeta = $x + 10$
10 years later age of Reeta = $y + 10$

A/Q
$$x + 10 = 2(y + 10)$$

Or, $x - 2y - 10 = 0 - - - - - - - - (2)$

Subtracting equation (1) from equation (2) we get

$$\begin{array}{r} x - 2y - 10 = 0 \\ -x \mp 3y \pm 10 = 0 \\ \hline y - 20 = 0 \end{array}$$

$\therefore\ y = 20$

Putting y = 20 in equation (2) we get

x - 2 x 20 - 10 - 0

x - 50 = 0

∴ x = 50

∴ Age of Neeta = 50 years and age of Reeta = 20 years

Example 15. Three chairs and two tables cost Rs 1850. Five chairs and three tables cost Rs 2850. Find the cost of two chairs and two tables.

Solutions:- Let price of one chair be Rs. x and price of one table be Rs. y.

· Price of 3 chairs and 2 tables is 3x + 2y

and price of 5 chairs and 3 tables is 5x + 3y

$A/Q\ \ 5x + 3y = 2850 - - - - - - - - (1)$

and 3x + 2y = 1850 ------------------- (2)

Multiplying equation (1) be 2 and equation (2) by 3 and then subtracting (2) from (1) we get

$$\begin{array}{rcr} 10x + 6y & = & 5700 \\ \_9x \pm 6y & = & \_5550 \\ \hline x & = & 150 \end{array}$$

Putting x = 150 in equation (2) we get

$$\begin{array}{l} 3 \times 150 + 2y = 1850 \\ Or, \quad 2y = 1850 - 450 \\ Or, \quad y = 700 \end{array}$$

·Cost of 2 chairs and 2 tables

$$2x + 2y = 2\times150 + 2\times700$$
$$Rs = 1700$$

Example 16. The sum of two digit number and the number obtained by reversing the order of its digit is 99. If the digits differ by 3, find the number.

Solution: Let the digit at unit's place be x and that at ten's place be y .

· Original number = 10y + x

Reversed number = 10x + y

$$A/Q \quad 10y + x + 10x + y = 99$$
$$Or, \quad 11x + 11y = 99$$
$$Or, \quad x + y = 9 ------------ (1)$$

Also, $x - y = \pm 3$

When x - y = 3 ---------------- (2)

and x + y = 9

Adding we get 2x = 12

$$\therefore x = 6$$

From (1) we get y = 3

· Original number = 10y + x

= 36

When x - y = -3 ------------------ (3)

Adding (1) and (3) we get $2x = 6 \quad \therefore x = 3$

and from (1) we get y = 6

· Original number = 10y + x

= 63

Hence the number is 36 or 63.

Example 17. A person can row downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find man's speed of rowing is still water and the speed of the current.

Solution:- Let man's speed of rowing be x km/h and speed of the current y km/h

$\therefore$ Down stream speed = x + y
and upstream speed = x - y

$A/Q \quad (x+y)\times 2 = 20$

Or, x + y = 10 -------------------(1)

and (x - y) x 2 = 4

Or, x - y = 2 ---------------------(2)

Adding (1) and (2) we get 2x = 12 Or, x = 6

Putting x = 6 in (1) we get 6 + y = 10 Or y = 4

$\therefore$ Speed of man =6km/h and speed of current =4km/h

Example 18. The sum of the munerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Determine the fraction.

Solution: - Let numerator be x and denominator be y

$$\therefore Fraction \ = \frac{x}{y}$$

A/Q x + y = 2y - 3

or, x - y + 3 = 0 -------------(1)

and x - 1 = 1/2(y - 1)

or, 2x - y - 1 = 0 ---- -----(2)

Subtracting equation (1) form (2) we get

$$\begin{array}{r} 2x - \ y - 1 = 0 \\ \_ x \mp y \pm 3 = 0 \\ \hline x - 4 = 0 \end{array}$$

$\therefore$ x = 4

From (1) 4 - y + 3 = 0

y = 7

$\therefore$ Fraction = x/y = 4/7

Example 19. Two places A and B are 120 km apart from each other on a highway. A car starts from A B and another from B at the same time. If they move in the same direction, they meet in 6 hours and if they move in opposite directions, they meet in 1 hour and 12 minutes. Find the speeds of the cars.

Solution: - Let the speed of car at A be X km/h and speed of car at B be y km/h

Distance covered by car at A in 6 hours = 6x

and distance covered by car at B in 6 hours = 6y

$$\therefore\ 6x - 6y = 120$$
$$or,\quad x - y = 20 - - - - - - - - -(1)$$

$$1\text{hour } 12\text{ minutes } = 1\frac{12}{60} = 1\frac{1}{5}$$
$$= \frac{6}{5}\ hour$$

$$\therefore\ \frac{6}{5}x + \frac{6}{5}y = 120$$

or, x + y = 100 ------------------- (2)

Adding (1) and (2) we get

$\therefore$ 2x = 120

$\because$ x = 60

Putting x = 60 in (2) we get y = 40

$\therefore$ Speeds of car are 60 km/h & 40 km/h.

Example – 21. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then . Also, three years from now, I shall be three times as old as you will be." Represent this situation algebraically and graphically.

Solution :- Let the present age of Aftab be x and that of his daughter y.

7 years ago, age of Aftab = x - 7 and age of his daughter = y - 7.

According to question, x - 7 = 7(y - 7)

Or, x - 7y + 42 = 0 --- --- --- --- (1)

After 3 years, age of Aftab = x + 3 and age of his daughter = y + 3.

According to question, x + 3 = 3(y + 3)

Or, x – 3y – 6 = 0 --- --- --- --- (2)

The above statement represented algebraically are (1) and (2).

To represent graphically, draw the graph. The points are given below:

For equation (1) :-&

x	0	7	14
y	6	7	8

For equation (2) :-

x	6	9	12
y	0	1	2

Exercise – 3

1. If twice the son's age in years is added to the age of his father the sum is 90. If twice the father's age in year is added to the age of the son, the sun in 120. Find their ages.

2. Ram is three times as old as Rahim. Five years later, Ram will be two-and-a –half times as old as Rahim. How old are Ram and Rahim now?

3. If the numerator of a fraction is multiplied by 2 and its denominator is increased by 2, it becomes 6/7. If instead we multiply the denominator by 2 and increase the numerator by 2 it reduces to 1/2. What is the fraction?

4. A fraction becomes 4/5 if 1 is added to each of the numerator and the denominator. However, if we subtract 5 from each, the fraction becomes 1/2 find the fraction.

5. If we add 1 in the numerator of a fraction and subtract 1 from its denominator, the fraction becomes 1, It is also given that the fraction becomes 1/2 when we add 1 to its denominator, and then what is the fraction.

6. If we add 5 to the denominator and subtract 5 from the numerator of fraction, it reduces to 1/7 if we subtract 3 from the numerator and add 3 to its denominator it reduces to 1/3. Find the fraction.

7. The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6, then the denominator becomes 12 times the numerator. Determine the fraction.

8. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator if the numerator and denominator are increased by 3, they are is the ratio 2 : 3. Determine the fraction.

9. Two audio cassettes and three video cassettes cost Rs. 340. but three audio cassettes and two video cassettes cost Rs 260. Find the price of an audio cassettes and that of a video cassettes.

10. Mala purchased 5 chairs and 2 tables for Rs. 1625. Reshma purchased 2 chairs and 1 table for Rs. 750. Find the cost per chair and per table.

11. If we buy 2 tickets from station A to station B, and 3 tickets from station A to station C, we have to pay Rs 795. but 3 ticket from station A to B and 5 ticket from station A to C cost a total of Rs 1300. What is the fare from station A to B and that from station A to C?

12. Aman travels 370 km partly by train and partly by car if he covers 250 km by train and the rest by car, it takes him 4 hours but if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.

13. The area of a rectangle get reduced by 9 square unit, if its length is reduced by 5 unit and the breadth is increased by 3unit if we increase the length by 3 unit and the breadth by 2 units, then the area is increased by 67 square unit. Find the length and the breadth of the rectangle.

14. If in a rectangle, the length is increased and the breadth is reduced by 2 units each, the area is reduced by 28 square units. If the length is reduced by 1 unit, and breadth increased by 2 units, the area increases by 33 square units. Find the dimensions of the rectangle.

15. The area of a rectangle gets reduced by 80 sq. units if its length is reduced by 5 units and the breadth in increased by 2 units. If we increase the length by 10 units and decrease the breadth by 5 units, the area in creased by 50 square units. Find the length and breadth of the rectangle.

16. A person starts his job with a certain monthly salary and earns a find increment every year. If his salary was Rs. 4500 after 4 years of service and Rs. 5400 after. 10 years of service, find his initial salary and the annual increment.

17. taxi charges consists of fixed charges and the remaining depending upon the distance traveled 70 km, he pay s Rs. 500 and for traveling 100 km, he pays Rs 680 express the above statements with the help of simultaneous equations and hence find the fixed charges and the rate per km.

18. The total expenditure per month of a house hold consists of a fixed rent of the house and the mess charge depending upon the number of people sharing the house. The total monthly expenditure is Rs. 3,900 for 2 people and Rs. 7,500 for 5 people. Find the rent of the house and the mess charges per head per month.

19. A railway half- ticket costs half the full fare but the reservation charges are the same on a half- ticket as on a full ticket one reserved first class ticket from station A to station B costs Rs. 2125. Also, one reserved first class ticket and one reserved half first class ticket from A to B cost Rs. 3200. find the full fare from station A to B and also the reservation charges for a ticket.

20. The sum of the digits of a two- digit number is 8. The number obtained by inter changing the two digits exceeds the given number by 36. find the number.

21. The sum of the digits of a two digits number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits of the number. Find the number.

22. Seven times a two digits number is the same as four times the number obtained on interchanging the digits of the given number. If one digit of the given number exceeds the other by 3, find the number.

23. A two digit number is obtained by either multiplying the sum of the digits by 8 and adding 1, or by multiplying the difference of the digits by 13 and adding 2. Find the number. How many such numbers are there?

24. A two- digits number is 4 times the sum of its digits. If 18 is added to the number, the digits are revered. Find the number.

25. A two- digits number is 4 more than 6 times the sum of its digits. If 18 is subtracted from the number the digits are reversed. Find the number.

26. The sum of a two digit number and the number formed by interchanging its digits is 110. if 10 is subtracted from the first number, the new number is 4 more than 5 times the sum of the digits in the first number. find the first number.

27. The sum of a two digits number and the number formed by interchanging the digits is 132. If 12 is added to the number, the new number becomes 5 times the sum of the digits. Find the number.

28. A number consists of two digits is seven times he sum of its digits. When 27 is subtracted from the number, the digits are reversed. Find the number.

29. A number consists of two digits. When it is divided by the sum of the digits, the quotient is 7. If 27 is subtracted from the number, the digits are reversed. Find the number.

30. A number consists of two digits when it is divided by sum of the digits, the quotient is 6 with no remainder. When the number is diminished by 9, the digits are reversed. Find the number.

31. In a triangle $ABC, \angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

32. In a cycle quadrilateral $ABCD, \angle A = (2x+4)^0, \angle B = (y+3)^0, \angle C = (2y+10)^0$ and $\angle D = (4x-5)^0$ find the four angles.

33. Places A and B are 80 km apart from each other on a highway. A car starts from A and another from B at the same time . If they moves in the same direction, they meet in 8 hours and if they move in opposite directions, they meet in 1 hours and 20 minutes. Find the speed of the cars.

34. Points A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the car travel in the same direction at a constant speed, they meet in 5 hours if the car travel towards each other, they meet in 1 hour. What are the speeds of the two cars.

35. A boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream is the same time. Find the speed of the boat in still water and the speed of the stream.

36. A boat goes 16 km upstream and 24 km down stream in 6 hours. It can go 12km upstream and 36km downstream in the same time. Find the speed of the boat in still water and the speed of the stream.

37. A Person can row 8 km upstream and 24 km down stream in 4 hours. He can row 12 km downstream and 12 km upstream in4 hours. Find the speed of the person in still water and also the speed of the current.

38. There are two class rooms A and B containing students. If 5 students are shifted from room A to room B, the resulting number of students in the two rooms become equal. If 5 student are shifted from room B to room A, the resulting number of student's in room A becomes double the number of student left in room B. find the original number of student in the two rooms seperately.

39. The coach of a cricket team buys three bats and six balls for Rs. 3900. Later, she buys another bat and two more balls of the same kind for Rs. 1300. Represent this situation algebraically and geometrically(graphically).
[$x + 2y = 1300$ and $x + 3y = 1300$, where x = cost in Rs. of one ball and y = cost in Rs. of bat].

40. The cost of 2 kg of apples and 1 kg of grapes on a day was found to be Rs 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically(graphically).
[$2x + y = 160$ and $4x + 2y = 300$, where x = price of 1 kg (in Rs.) of apple and y = price of 1 kg (in Rs.) of grapes].

41. Akhila went to a fair with Rs. 20 and want to have rides on the Giant Wheel and play Hoopla. The number of times she played Hoopla is half the number of rides she had on the Giant Wheel. If each ride costs Rs. 3 and a game of Hoopla costs Rs. 4, how would you find out the number of rides she had and how many times she played Hoopla.
[$x – 2y = 0$ and $3x + 4y = 20$; the value of $x = 4$ and $y = 2$].

42. Romila went to a stationary shop and purchased 2 pencils and 3 erasers for Rs. 9. Her friend Sonali saw the new variety of pencils and erasers with Romila, and she also bought 4 pencils and 6 erasers of the same kind for Rs. 18. Represent this situation algebraically and graphically.
[$2x + 3y = 9$ and $4x + 6y = 18$].

43. Two rails are represented by the equations $x + 2y – 4 = 0$ and $2x + 4y – 12 = 0$. Represent this situation geometrically.

44. Champa went to a ‘Sale’ to purchase some pants and skirts. When her friends asked

her how many of each she had bought, she answered, “The number of skirts is two less than twice the number of paints purchased. Also, the number of skirts is four less than four times the number of pants purchased”. Help her friend to find how many pants and skirts Champa bought.
[y = 2x – 2 and y = 4x – 4 , where x = no. of pants and y = no. of skirts].

Answer

(1) 20 yrs, 50 yrs

(2) Ram: 45, rahim: 15

(4) 7/9

(5) 3/5

(7) 7/18

(8) 5/9

(10) Rs. 125, Rs. 500

(11) Rs. 75, Rs. 215

(13) 17 units, 9 units

(14) 23 units, 11 units

(16) Rs. 3900, Rs. 150

(17) Rs. 80, Rs. 6 per km

(19) Rs. 2100, Rs. 25

(20) 26

(22) 36

(23) 41,one

(25) 64

(26) 64

(28) 63

(29) 63

(31) 20^0, 40^0, 120^0

(32) 70^0, 53^0, 110^0, 127^0

(34) 60 km/h, 40 km/h

(35) 6 km/h, 2 km/h

(37) 8 km/h, 4 km/h

(38) A: 35, B: 25