APPENDIX IV

Logarithms

Sometimes, a numerical expression may involve multiplication, division or rational powers of large numbers. For such calculations, logarithms are very useful. They help us in making difficult calculations easy. In Chemistry, logarithm values are required in solving problems of chemical kinetics, thermodynamics, electrochemistry, etc. We shall first introduce this concept, and discuss the laws, which will have to be followed in working with logarithms, and then apply this technique to a number of problems to show how it makes difficult calculations simple.

We know that

$2^3 = 8,\ 3^2 = 9,\ 5^3 = 125,\ 7^0 = 1$

In general, for a positive real number a, and a rational number m, let $a^m = b$,

where b is a real number. In other words

the m^{th} power of base a is b.

Another way of stating the same fact is

logarithm of b to base a is m.

If for a positive real number a, $a \neq 1$

$a^m = b$,

we say that m is the logarithm of b to the base a.

We write this as $\log{}_a^b = m$,

"log" being the abbreviation of the word "logarithm".

Thus, we have

$\log_2 8 = 3,$ Since $2^3 = 8$

$\log_3 9 = 2,$ Since $3^2 = 9$

$\log \frac{125}{5} = 3,$ Since $5^3 = 125$

$\log_7 1 = 0,$ Since $7^0 = 1$

Laws of Logarithms

In the following discussion, we shall take logarithms to any base a, ($a > 0$ and $a \neq 1$)

First Law: $\log_a (mn) = \log_a m + \log_a n$

Proof: Suppose that $\log_a m = x$ and $\log_a n = y$

Then $a^x = m,\ a^y = n$

Hence $mn = a^x.a^y = a^{x+y}$

It now follows from the definition of logarithms that

$\log_a (mn) = x + y = \log_a m - \log_a n$

Second Law: $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

Proof: Let $\log_a m = x,\ \log_a n = y$

Then $a^x = m,\ a^y = n$

Hence $\frac{m}{n} = \frac{a^x}{a^y} = a^{x-y}$

Therefore

$$\log_a \left(\frac{m}{n}\right) = x - y = \log_a m - \log_a n$$

Third Law : $\log_a(m^n) = n \log_a m$

Proof : As before, if $\log_a m = x$, then $a^x = m$

Then $m^n = \left(a^x\right)^n = a^{nx}$

giving $\log_a(m^n) = nx = n \log_a m$

Thus according to First Law: "the log of the product of two numbers is equal to the sum of their logs. Similarly, the Second Law says: the log of the ratio of two numbers is the difference of their logs. Thus, the use of these laws converts a problem of multiplication / division into a problem of addition/ subtraction, which are far easier to perform than multiplication/division. That is why logarithms are so useful in all numerical computations.

Logarithms to Base 10

Because number 10 is the base of writing numbers, it is very convenient to use logarithms to the base 10. Some examples are:

$\log_{10} 10 = 1$, since $10^1 = 10$

$\log_{10} 100 = 2$, since $10^2 = 100$

$\log_{10} 10000 = 4$, since $10^4 = 10000$

$\log_{10} 0.01 = -2$, since $10^{-2} = 0.01$

$\log_{10} 0.001 = -3$, since $10^{-3} = 0.001$

and $\log_{10} 1 = 0$ since $10^0 = 1$

The above results indicate that if n is an integral power of 10, i.e., 1 followed by several zeros or 1 preceded by several zeros immediately to the right of the decimal point, then log n can be easily found.

If n is not an integral power of 10, then it is not easy to calculate log n. But mathematicians have made tables from which we can read off approximate value of the logarithm of any positive number between 1 and 10. And these are sufficient for us to calculate the logarithm of any number expressed in decimal form. For this purpose, we always express the given decimal as the product of an integral power of 10 and a number between 1 and 10.

Standard Form of Decimal

We can express any number in decimal form, as the product of (i) an integral power of 10, and (ii) a number between 1 and 10. Here are some examples:

(i) 25.2 lies between 10 and 100

$$25.2 = \frac{25.2}{10} \times 10 = 2.52 \times 10^1$$

(ii) 1038.4 lies between 1000 and 10000.

$$\therefore 1038.4 = \frac{1038.4}{1000} \times 10^3 = 1.0384 \times 10^3$$

(iii) 0.005 lies between 0.001 and 0.01

$\therefore$ $0.005 = (0.005 \times 1000) \times 10^{-3} = 5.0 \times 10^{-3}$

(iv) 0.00025 lies between 0.0001 and 0.001

$\therefore$ $0.00025 = (0.00025 \times 10000) \times 10^{-4} = 2.5 \times 10^{-4}$

In each case, we divide or multiply the decimal by a power of 10, to bring one non-zero digit to the left of the decimal point, and do the reverse operation by the same power of 10, indicated separately.

Thus, any positive decimal can be written in the form

$n = m \times 10^p$

where p is an integer (positive, zero or negative) and $1 \leq m < 10$. This is called the "standard form of n."

Working Rule

1. Move the decimal point to the left, or to the right, as may be necessary, to bring one non-zero digit to the left of decimal point.
2. (i) If you move p places to the left, multiply by 10^p.
   (ii) If you move p places to the right, multiply by 10^{-p}.
   (iii) If you do not move the decimal point at all, multiply by 10^0.
   (iv) Write the new decimal obtained by the power of 10 (of step 2) to obtain the standard form of the given decimal.

Characteristic and Mantissa

Consider the standard form of n

$n = m \times 10^p$, where $1 \leq m < 10$

Taking logarithms to the base 10 and using the laws of logarithms

$\log n = \log m + \log 10^p$

$= \log m + p \log 10$

$= p + \log m$

Here p is an integer and as $1 \leq m < 10$, so $0 \leq \log m < 1$, i.e., m lies between 0 and 1. When log n has been expressed as p + log m, where p is an integer and 0 log m < 1, we say that p is the "characteristic" of log n and that log m is the "mantissa of log n. Note that characteristic is always an integer – positive, negative or zero, and mantissa is never negative and is always less than 1. If we can find the characteristics and the mantissa of log n, we have to just add them to get log n.

Thus to find log n, all we have to do is as follows:

1. Put n in the standard form, say
   $n = m \times 10^p$, $1 \leq m < 10$
2. Read off the characteristic p of log n from this expression (exponent of 10).
3. Look up log m from tables, which is being explained below.
4. Write log n = p + log m

If the characteristic p of a number n is say, 2 and the mantissa is .4133, then we have log n = 2 + .4133 which we can write as 2.4133. If, however, the characteristic p of a number m is say –2 and the mantissa is .4123, then we have log m = –2 + .4123. We cannot write this as –2.4123. (Why?) In order to avoid this confusion we write $\bar{2}$ for –2 and thus we write log m = $\bar{2}.4123$.

Now let us explain how to use the table of logarithms to find mantissas. A table is appended at the end of this Appendix.

Observe that in the table, every row starts with a two digit number, 10, 11, 12,... 97, 98, 99. Every column is headed by a one-digit number, 0, 1, 2, ...9. On the right, we have the section called "Mean differences" which has 9 columns headed by 1, 2...9.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
61	7853	7860	7868	7875	7882	7889	7896	7803	7810	7817	1	1	2	3	4	4	5	6	6
62	7924	7931	7935	$\boxed{7945}$	7954	7959	7966	7973	7980	7987	1	1	2	$\boxed{3}$	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	6	6

Now suppose we wish to find log (6.234). Then look into the row starting with 62. In this row; look at the number in the column headed by 3. The number is 7945. This means that

log (6.230) = 0.7945*

But we want log (6.234). So our answer will be a little more than 0.7945. How much more? We look this up in the section on Mean differences. Since our fourth digit is 4, look under the column headed by 4 in the Mean difference section (in the row 62). We see the number 3 there. So add 3 to 7945. We get 7948. So we finally have

log (6.234) = 0.7948.

Take another example. To find log (8.127), we look in the row 81 under column 2, and we find 9096. We continue in the same row and see that the mean difference under 7 is 4. Adding this to 9096, and we get 9100. So, log (8.127) = 0.9100.

Finding *N* when log *N* is given

We have so far discussed the procedure for finding log n when a positive number n given. We now turn to its converse i.e., to find n when log n is given and give a method for this purpose. If log n = t, we sometimes say n = antilog t. Therefore our task is given t, find its antilog. For this, we use the ready-made antilog tables.

Suppose log n = 2.5372.

To find n, first take just the mantissa of log n. In this case it is .5372. (Make sure it is positive.) Now take up antilog of this number in the antilog table which is to be used exactly like the log table. In the antilog table, the entry under column 7 in the row .53 is 3443 and the mean difference for the last digit 2 in that row is 2, so the table gives 3445. Hence,

antilog (.5372) = 3.445

Now since log n = 2.5372, the characteristic of log n is 2. So the standard form of n is given by

$n = 3.445 \times 10^2$

or n = 344.5

Illustration 1:

If log x = 1.0712, find x.

Solution: We find that the number corresponding to 0712 is 1179. Since characteristic of log *x* is 1, we have

$x = 1.179 \times 10^1$

$= 11.79$

Illustration 2:

If log x = $\bar{2}.1352$, find x.

Solution: From antilog tables, we find that the number corresponding to 1352 is 1366. Since the characteristic is $\bar{2}$ i.e., –2, so

$x = 1.366 \times 10^{-2} = 0.01366$

Use of Logarithms in Numerical Calculations

Illustration 1:

Find 6.3 × 1.29

Solution: Let x = 6.3 × 1.29

Then log x = log (6.3 × 1.29) = log 6.3 + log 1.29

Now,

log 6.3 = 0.7993

log 1.29 = 0.1106

∴ log x = 0.9099,

Taking antilog

* *It should, however, be noted that the values given in the table are not exact. They are only approximate values, although we use the sign of equality which may give the impression that they are exact values. The same convention will be followed in respect of antilogarithm of a number.*

$x = 8.127$

Illustration 2:

Find $\dfrac{(1.23)^{1.5}}{11.2 \times 23.5}$

Solution: Let $x = \dfrac{(1.23)^{\frac{3}{2}}}{11.2 \times 23.5}$

Then $\log x = \log \dfrac{(1.23)^{\frac{3}{2}}}{11.2 \times 23.5}$

$= \dfrac{3}{2} \log 1.23 - \log (11.2 \times 23.5)$

$= \dfrac{3}{2} \log 1.23 - \log 11.2 - 23.5$

Now,

$\log 1.23 = 0.0899$

$\dfrac{3}{2} \log 1.23 = 0.13485$

$\log 11.2 = 1.0492$

$\log 23.5 = 1.3711$

$\log x = 0.13485 - 1.0492 - 1.3711$

$= \bar{3}.71455$

$\therefore x = 0.005183$

Illustration 3:

Find $\sqrt{\dfrac{(71.24)^5 \times \sqrt{56}}{(2.3)^7 \times \sqrt{21}}}$

Solution: Let $x = \sqrt{\dfrac{(71.24)^5 \times \sqrt{56}}{(2.3)^7 \times \sqrt{21}}}$

Then $\log x = \dfrac{1}{2} \log \left[\dfrac{(71.24)^5 \times \sqrt{56}}{(2.3)^7 \times \sqrt{21}}\right]$

$= \dfrac{1}{2} [\log (71.24)^5 + \log \sqrt{56} - \log (2.3)^7 - \log \sqrt{21}]$

$= \dfrac{5}{2} \log 71.24 + \dfrac{1}{4} \log 56 - \dfrac{7}{2} \log 2.3 - \dfrac{1}{4} \log 21$

Now, using log tables

$\log 71.24 = 1.8527$

$\log 56 = 1.7482$

$\log 2.3 = 0.3617$

$\log 21 = 1.3222$

$\therefore \log x = \dfrac{5}{2} \log (1.8527) + \dfrac{1}{4} (1.7482) - \dfrac{7}{2} (0.3617) - \dfrac{1}{4} (1.3222)$

$= 3.4723$

$\therefore x = 2967$