CHAPTER - 3

TRIGONOMETRIC FUNCTIONS

KEY POINTS

- A radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle. We denote 1 radian by 1^c.
- π radian = 180 degree

  1 radian = $\frac{180}{\pi}$ degree

  1 degree = $\frac{\pi}{180}$ radian
- If an arc of length l makes an angle θ radian at the centre of a circle of radius r, we have

  $\theta = \frac{l}{r}$

Quadrant →	I	II	III	IV
t- functions which are positive	All	sin x cosec x	tan x cot x	cos x sec x

Function	–x	$\frac{\pi}{2} - x$	$\frac{\pi}{2} + x$	$\pi - x$	$\pi + x$	$2\pi - x$	$2\pi + x$
sin	–sin x	cos x	cos x	sin x	–sin x	–sin x	sin x
cos	cos x	sin x	–sin x	–cos x	–cos x	cos x	cos x
tan	–tan x	cot x	–cot x	–tan x	tan x	–tan x	tan x
cosec	–cosec x	sec x	sec x	cosec x	–cosec x	–cosec x	cosec x
sec	sec x	cosec x	–cosec x	–sec x	–sec x	sec x	sec x
cot	–cot x	tan x	–tan x	–cot x	cot x	–cot x	cot x

Function	Domain	Range
sin x	R	[–1,1]
cos x	R	[–1,1]
tan x	$R - \left\{(2n+1)\frac{\pi}{2}; n \in z\right\}$	R
Cosec x	R – {nπ; n ∈ z}	R – (–1,1)
Sec x	$R - \left\{(2n+1)\frac{\pi}{2}; n \in z\right\}$	R – (–1,1)
cot x	R – {nπ, n ∈ z}	R

Some Standard Results

- sin (x + y) = sinx cosy + cosx siny

  cos (x + y) = cosx cosy – sinx siny

  $$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x.\tan y}$$

  $$\cot(x+y) = \frac{\cot x.\cot y - 1}{\cot y + \cot x}$$

- sin (x – y) = sinx cosy – cosx siny

  cos (x – y) = cosx cosy + sinx siny

  $$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x.\tan y}$$

  $$\cot(x-y) = \frac{\cot x.\cot y + 1}{\cot y - \cot x}$$

- $$\tan(x+y+z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y.\tan z - \tan z \tan x}$$

- 2sinx cosy = sin(x + y) + sin(x – y)

  2cosx siny = sin(x + y) – sin(x – y)

  2cosx cosy = cos(x + y) + cos(x – y)

  2sinx siny = cos(x – y) – cos(x + y)

- $\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$

  $\sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2}$

  $\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$

  $\cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}$

- $\text{Sin } 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$

- $\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$

- $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

- $\sin 3x = 3 \sin x - 4 \sin^3 x$

- $\cos 3x = 4 \cos^3 x - 3 \cos x$

- $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

- $\sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y$

  $= \cos^2 y - \cos^2 x$

- $\cos(x + y) \cos(x - y) = \cos^2 x - \sin^2 y$

  $= \cos^2 y - \sin^2 x$

- Principal solutions – The solutions of a trigonometric equation for which $0 \leq x < 2\pi$ are called its principal solutions.

- General solution – A solution of a trigonometric equation, generalised by means of periodicity, is known as the general solution.

General solutions of trigonometric equations :

$\sin \theta = 0 \Rightarrow \theta = n\pi, n \in z$

$\cos \theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}, n \in z$

$\tan \theta = 0 \Rightarrow \theta = n\pi, n \in z$

$\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in z$

$\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in z$

$\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha, n \in z$

- Law of sines or sine formula

  The lengths of sides of a triangle are proportional to the sines of the angles opposite to them i.e..

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- Law of cosines or cosine formula

  In any Δ ABC

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find the radian measure corresponding to 5° 37' 30"

2. Find the degree measure corresponding to $\left(\frac{11}{16}\right)^c$

3. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°

4. Find the value of $\tan\frac{19\pi}{3}$

5. Find the value of $\sin(-1125°)$

6. Find the value of $\tan 15°$

7. If $\sin A = \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$, find $\cos A$

8. If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$ then find the value of $A + B$.

9. Express $\sin 12\theta + \sin 4\theta$ as the product of sines and cosines.

10. Express $2\cos 4x \sin 2x$ as an algebraic sum of sines or cosines.

11. Write the range of $\cos\theta$

12. What is domain of $\sec\theta$?

13. Find the principal solutions of $\cot x = -\sqrt{3}$

14. Write the general solution of $\cos\theta = 0$

15. If $\sin x = \frac{\sqrt{5}}{3}$ and $0 < x < \frac{\pi}{2}$ find the value of $\cos 2x$

16. If $\cos x = \frac{-1}{3}$ and x lies in quadrant III, find the value of $\sin\frac{x}{2}$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

17. A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 metres when it traces 72° at the centre, find the length of the rope.

18. It the angles of a triangle are in the ratio 3:4:5, find the smallest angle in degrees and the greatest angle in radians.

19. If $\sin x = \frac{12}{13}$ and x lies in the second quadrant, show that $\sec x + \tan x = -5$

20. If $\cot \alpha = \frac{1}{2}$, $\sec \beta = \frac{-5}{3}$ where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, find the value of $\tan(\alpha + \beta)$

Prove the following Identities

21. $\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4 \cos 2\theta \cos 4\theta$

22. $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2 \tan 2x$

23. $\frac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x \sin x + \cos 6x \cos x} = \tan 2x$

24. $\frac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan\frac{\theta}{2}$

25. $\tan \alpha . \tan(60° - \alpha) . \tan(60° + \alpha) = \tan 3\alpha$

26. Show that $\cos 20° \cos 40° \cos 80° = \frac{1}{8}$

27. Show that $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2\cos\theta$

28. Pr ove that $\frac{\cos x}{1 - \sin x} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$

29. Draw the graph of cosx in $[0, 2\pi]$

Find the general solution of the following equations (Q.No. 30 to Q. No. 33)

30. $\cos\left(x + \frac{\pi}{10}\right) = 0$

31. $\sin 7x = \sin 3x$

32. $\sqrt{3}\cos x - \sin x = 1$

33. $3 \tan x + \cot x = 5 \operatorname{cosec} x$

34. In any triangle ABC, prove that

$$a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B) = 0$$

35. In any triangle ABC, prove that

$$a = b\cos C + c\cos B$$

36. In any triangle ABC, prove that

$$\frac{a+b}{c} = \frac{\cos\frac{A-B}{2}}{\sin\frac{C}{2}}$$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

37. Prove that

$$\cos A\ \cos 2A\ \cos 4A\ \cos 8A = \frac{\sin 16A}{16\ \sin A}$$

38. Prove that $\sin 10^\circ\ \sin 30^\circ\ \sin 50^\circ\ \sin 70^\circ = \frac{1}{16}$

39. Find the general solution of

$$\sin 2x + \sin 4x + \sin 6x = 0$$

40. Find the general solution of

$$\cos\theta\ \cos 2\theta\ \cos 3\theta = \frac{1}{4}$$

41. Draw the graph of tanx in $\left(\frac{-3\pi}{2}, \frac{3\pi}{2}\right)$

42. In any triangle ABC, prove that

$$\frac{b^2 - c^2}{a^2}\sin 2A + \frac{c^2 - a^2}{b^2}\sin 2B + \frac{\left(a^2 - b^2\right)}{c^2}\sin 2C = 0$$

ANSWERS

1. $\left(\frac{\pi}{32}\right)^c$

2. 39°22'30"

3. $\frac{5\pi}{12}$ cm

4. $\sqrt{3}$

5. $\frac{-1}{\sqrt{2}}$

6. $2 - \sqrt{3}$

7. $\frac{-4}{5}$

8. 45°

9. $2 \sin 8\theta \cos 4\theta$

10. $\sin 6x - \sin 2x$

11. $[-1,1]$

12. $R - \left\{(2n+1)\frac{\pi}{2}; n \in z\right\}$

13. $\frac{5\pi}{6}, \frac{11\pi}{6}$

14. $(2n+1)\frac{\pi}{2}, n \in z$

15. $-\frac{1}{9}$

16. $\frac{\sqrt{6}}{3}$

17. 70 m

18. $45°, \frac{5\pi}{12}$ radians

20 $\frac{2}{11}$

30. $\left(n\pi + \frac{2\pi}{5}\right), n \in z$

31. $(2n+1)\frac{\pi}{10}, \frac{n\pi}{2}, n \in z$

32. $2n\pi \pm \frac{\pi}{3} - \frac{\pi}{6}, n \in z$

33. $2n\pi \pm \frac{\pi}{3}, n \in z$

39. $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}, n \in z$

40. $(2n+1)\frac{\pi}{8}, n\pi \pm \frac{\pi}{3}, n \in z$