CHAPTER - 4

PRINCIPLE OF MATHEMATICAL INDUCTION

KEY POINTS

- Induction and deduction are two basic processes of reasoning.
- Deduction is the application of a general case to a particular case. In contrast to deduction, induction is process of reasoning from particular to general.
- Principle of Mathematical Induction :

  Let P(n) be any statement involving natural number n such that

  (i) P(1) is true, and

  (ii) If P(k) is true implies that P(k +1) is also true for some natural number k

  then P(n) is true $\forall n \in N$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

Using the principle of mathematical induction prove the following for all $n \in N$:

1. $3.6 + 6.9 + 9.12 + ... + 3n(3n + 3) = 3n(n + 1)(n + 2)$

2. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)---\left(1-\frac{1}{n+1}\right)=\frac{1}{n+1}$

3. $n^2 + n$ is an even natural number.

4. $2^{3n} - 1$ is divisible by 7

5. 3^{2n} when divided by 8 leaves the remainder 1.

6. $4^n + 15n - 1$ is divisible by 9

7. $n^3 + (n + 1)^3 + (n + 2)^3$ is a multiple of 9.

8. $x^{2n-1} - 1$ is divisible by $x - 1$, $x \neq 1$

9. $3^n > n$

10. If x and y are any two distinct integers then $x^n - y^n$ is divisible by $(x - y)$

11. $n < 2^n$

12. $a + (a + d) + (a + 2d) + \dots\dots\dots + [a + (n - 1)d] = \frac{n}{2}[2a + (n - 1)d]$

13. $3x + 6x + 9x + \dots\dots\dots$ to n terms $= \frac{3}{2}n(n + 1)x$

14. $11^{n+2} + 12^{2n+1}$ is divisible by 133.