TOPIC 2
INVERSE TRIGONOMETRIC FUNCTIONS
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	References NCERT Text Book XI Ed. 2007
Inverse Trigonometric Functions	(i).Principal value branch Table	**	Ex 2.1 QNo- 11, 14
	(ii). Properties of Inverse Trigonometric Functions	***	Ex 2.2 Q No- 7,13, 15 Misc Ex Q.No. 9,10,11,12

SOME IMPORTANT RESULTS/CONCEPTS

* Domain & Range of the Inverse Trigonometric Function:

	Functions	Domain	Range (Principal value Branch)
i.	$\sin^{-1}$:	$[-1,1]$	$[-\pi/2, \pi/2]$
ii.	$\cos^{-1}$:	$[-1,1]$	$[0,\pi]$
iii.	$\operatorname{cosec}^{-1}$:	$R-(-1,1)$	$[-\pi/2,\pi/2]-\{0\}$
iv.	$\sec^{-1}$:	$R-(-1,1)$	$[0,\pi]-\{\pi/2\}$
v.	$\tan^{-1}$:	R	$(-\pi/2,\pi/2)$
vi.	$\cot^{-1}$:	R	$(0,\pi)$

* Properties of Inverse Trigonometric Function

1. (i) $\sin^{-1}(\sin x)=x$ & $\sin(\sin^{-1}x)=x$ ii. $\cos^{-1}(\cos x)=x$ & $\cos(\cos^{-1}x)=x$

   iii. $\tan^{-1}(\tan x)=x$ & $\tan(\tan^{-1}x)=x$ iv. $\cot^{-1}(\cot x)=x$ & $\cot(\cot^{-1}x)=x$

   v. $\sec^{-1}(\sec x)=x$ & $\sec(\sec^{-1}x)=x$ vi. $\operatorname{cosec}^{-1}(\operatorname{cosec} x)=x$ & $\operatorname{cosec}(\operatorname{cosec}^{-1}x)=x$

2. i. $\sin^{-1}x=\operatorname{cosec}^{-1}\frac{1}{x}$ & $\sin^{-1}x=\operatorname{cosec}^{-1}\frac{1}{x}$ ii. $\cos^{-1}x=\sec^{-1}\frac{1}{x}$ & $\sec^{-1}x=\cos^{-1}\frac{1}{x}$

   iii. $\tan^{-1}x=\cot^{-1}\frac{1}{x}$ & $\cot^{-1}x=\tan^{-1}\frac{1}{x}$

3. i $\sin^{-1}(-x)=-\sin^{-1}x$ iv $\cos^{-1}(-x)=\pi-\cos^{-1}x$

   ii. $\tan^{-1}(-x)=-\tan^{-1}x$ v $\sec^{-1}(-x)=\pi-\sec^{-1}x$

   iii. $\operatorname{cosec}^{-1}(-x)=-\operatorname{cosec}^{-1}x$ vi $\cot^{-1}(-x)=\pi-\cot^{-1}x$

4. i $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$ ii. $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$

   iii. $\operatorname{cosec}^{-1}x+\sec^{-1}x=\frac{\pi}{2}$

5. $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

6. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy < 1$

$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy > 1$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ if $xy > -1$

ASSIGNMENTS

(i). Principal value branch Table

LEVEL I

Write the principal value of the following :

1. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

2. $\sin^{-1}\left(-\frac{1}{2}\right)$

3. $\tan^{-1}\left(-\sqrt{3}\right)$

4. $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

LEVEL II

Write the principal value of the following :

1. $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ **[CBSE 2011]**

2. $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)$

3. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

(ii). Properties of Inverse Trigonometric Functions

LEVEL I

1. Evaluate $\cot[\tan^{-1}a + \cot^{-1}a]$

2. Prove $3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right)$

3. Find x if $\sec^{-1}\left(\sqrt{2}\right) + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

LEVEL II

1. Write the following in simplest form : $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x \neq 0$

2. Prove that $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

3. Prove that $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$.

4. Prove that $2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{31}{17}\right)$ **[CBSE 2011]**

5. Prove that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$ **[CBSE 2012]**

LEVEL III

1. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\frac{\pi}{4}\right)$

2. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$ **[CBSE 2011]**

3. Solve $\tan^{-1}2x+\tan^{-1}3x=\pi/4$

4. Solve $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

5.Solve $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

6. Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2},\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ **[CBSE 2012]**

Questions for self evaluation

1. Prove that $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{63}{16}$

2. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\qquad x\in\left[-\frac{1}{\sqrt{2}},1\right]$

3. Prove that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

4. Prove that $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$

5. Prove that $\tan^{-1}\left(\frac{x}{y}\right)-\tan\left(\frac{x-y}{x+y}\right)=\frac{\pi}{4}$

6. Write in the simplest form $\cos\left[2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right]$

7. Solve $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

8. Solve $\tan^{-1}2x+\tan^{-1}3x=\pi/4$