

1. Real numbers**(Key Points)**

- Euclid's Division lemma:-** Given Positive integers a and b there exist unique integers q and r satisfying $a = bq + r$, where $0 \leq r < b$, where a , b , q and r are respectively called as dividend, divisor, quotient and remainder.
- Euclid's division Algorithm:-** To obtain the HCF of two positive integers say c and d , with $c > 0$, follow the steps below:

Step I: Apply Euclid's division lemma, to c and d , so we find whole numbers, q and r such that $c = dq + r$, $0 \leq r < d$.

Step II: If $r = 0$, d is the HCF of c and d . If $r \neq 0$, apply the division lemma to d and r .

Step III: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF

- The Fundamental theorem of Arithmetic:-**

Every composite number can be expressed (factorised) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

Ex.: $24 = 2 \times 2 \times 2 \times 3 = 3 \times 2 \times 2 \times 2$

Theorem: Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form

Of $\frac{p}{q}$ where p and q are co-prime and the prime factorisation of q is the form of $2^n \cdot 5^m$, where n, m are non negative integers.

Ex. $\frac{7}{10} = \frac{7}{2 \times 5} = 0.7$

Theorem: Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of q is not of the form of $2^n \cdot 5^m$, where n, m are non negative integers. Then x has a decimal expansion which is non terminating repeating (recurring).

Ex. $\frac{7}{6} = \frac{7}{2 \times 3} = 1.1666 \dots$

Theorem: For any two positive integers a and b ,

$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$

Ex.: 4 & 6 ; $\text{HCF}(4, 6) = 2$, $\text{LCM}(4, 6) = 12$; $\text{HCF} \times \text{LCM} = 2 \times 12 = 24$

Ans. : $a \times b = 24$

(Level- 1)

- If $\frac{p}{q}$ is a rational number ($q \neq 0$). What is the condition on q so that the decimal representation of $\frac{p}{q}$ is terminating?
Ans. q is form of $2^n \cdot 5^m$ where n, m are non negative integers.
- Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.
Ans. 1.5
- The decimal expansion of the rational no. $\frac{43}{2^4 \cdot 5^3}$ will terminate after how many of decimals?
Ans. After 4 places of decimal.
- Find the $(\text{HCF} \times \text{LCM})$ for the numbers 100 and 190.

Ans. 19000

5. State whether the number $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})$ is rational or irrational justify.

Ans. Rational

6. Write one rational and one irrational number lying between 0.25 and 0.32.

Ans. One rational no. = 0.26, one irrational no. = 0.27010010001.....

7. Express 107 in the form of $4q + 3$ for some positive integer.

Ans. $4 \times 26 + 3$

8. Write whether the rational number $\frac{51}{1500}$ will have a terminating decimal expansion or a non terminating repeating decimal expansion.

Ans. Terminating.

(level - 2)

1. Use Euclid's division algorithm to find the HCF of 1288 and 575.

Ans. 23.

2. Check whether $5 \times 3 \times 11 + 11$ and $5 \times 7 + 7 \times 3 + 3$ are composite number and justify.

Ans. Composite number.

3. Check whether 6^n can end with the digit 0, where n is any natural number.

Ans. No, 6^n can not end with the digit 0.

4. Given that $\text{LCM}(26, 169) = 338$, write $\text{HCF}(26, 169)$.]

Ans. 13

5. Find the HCF and LCM of 6, 72 and 120 using the prime factorization method.

Ans. HCF = 6
LCM = 360**(level - 3)**

1. Show that $\sqrt{3}$ is an irrational number.

2. Show that $5 + 3\sqrt{2}$ is an irrational number.

3. Show that square of an odd positive integer is of the form $8m + 1$, for some integer m.

4. Find the LCM & HCF of 26 and 91 and verify that $\text{LCM} \times \text{HCF} = \text{product of the two numbers}$.

Ans. LCM=182, HCF=13

(PROBLEMS FOR SELF EVALUATION/HOTS)

1. State the fundamental theorem of Arithmetic.
2. Express 2658 as a product of its prime factors.
3. Show that the square of an odd positive integers is of the form $8m + 1$ for some whole number m.
4. Find the LCM and HCF of 17, 23 and 29.

5. Prove that $\sqrt{2}$ is not a rational number.
6. Find the largest positive integer that will divide 122, 150 and 115 leaving remainder 5, 7 and 11 respectively.
7. Show that there is no positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.
8. Using prime factorization method, find the HCF and LCM of 72, 126 and 168. Also show that $HCF \times LCM \neq \text{product of the three numbers}$.

2. Polynomials (Key Points)

Polynomial:

An expression of the form $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ where $a_n \neq 0$ is called a polynomial in variable x of degree n . where; $a_0, a_1, \dots, a_n$ are real numbers and each power of x is a non negative integer.

Ex.:- $2x^2 - 5x + 1$ is a polynomial of degree 2.

Note: $\sqrt{x} + 3$ is not a polynomial.

- A polynomial $p(x) = ax + b$ of degree 1 is called a linear polynomial. Ex. $5x - 3, 2x$ etc
- A polynomial $p(x) = ax^2 + bx + c$ of degree 2 is called a quadratic polynomial. Ex. $2x^2 + x - 1, 1 - 5x + x^2$ etc.
- A polynomial $p(x) = ax^3 + bx^2 + cx + d$ of degree 3 is called a cubic polynomial. Ex. $\sqrt{3}x^3 - x + \sqrt{5}, x^3 - 1$ etc.

Zeroes of a polynomial: A real number k is called a zero of polynomial $p(x)$ if $p(x) = 0$. The graph of $y = p(x)$ intersects the X-axis.

- A linear polynomial has only one zero.
- A Quadratic polynomial has two zeroes.
- A Cubic polynomial has three zeroes.

For a quadratic polynomial: If α, β are zeroes of $P(x) = ax^2 + bx + c$ then :

1. Sum of zeroes $= \alpha + \beta = \frac{-b}{a} = \frac{\text{Coefficient of } x}{\text{coefficient of } x^2}$
2. Product of zeroes $= \alpha \cdot \beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{coefficient of } x^2}$

- A quadratic polynomial whose zeroes are α and β , is given by:

$$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - (\text{sum of zeroes})x + \text{product of zeroes}.$$
- If α, β and γ are zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ then:

$$* \alpha + \beta + \gamma = \frac{-b}{a}$$

$$* \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$* \alpha\beta\gamma = \frac{-d}{a}$$

Division algorithm for polynomials: If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that: